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Path Union and Cycle of Graphs with Mean Labeling

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ABSTRACT

In this paper we investigate mean labeling for path union of $K_{2,m}$, P_n, $P_n\times P_m$, C_n. Also we prove that the mean labeling for cycle of P_n, C_n, $P_n\times P_m$. Path unions of any mean graph are mean graph for that were call Step grid graphics mean graph.

KEY WORDS: Cycle, Complete bipartite graph, Grid graph, Step grid graph, Path union of graphs, Cycle of graphs and mean labeling.

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1: INTRODUCTION

We begin with a simple, undirected and finite graph $G=(V,E)$with $|V|=p$vertices and$|E|=q$edges.Forallterminology,notationsandbasicdefinitionswefollowsHarary[1]. First of all we give brief summary of definitions which are used in this paper.

Definition−1.1: If the vertices of the graph are assigned values subject to certain conditions then it is known as *graph labeling*.

Definition−1.2:A function f is called *mean labeling* for a graph $G = (V, E)$ if $f : V \longrightarrow \{0, 1, . . . , q\}$is injective and the induce function $f^* : E \longrightarrow \{1, 2, . . . , q\}$ defined as $f^*(e) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is objective for every edge $e=(u,v)\in E$. Agraph G is called *mean graph* if it admits a mean labeling.

Definition−1.3:For a cycle C_n, each vertices of $C_n$is replace by connected graphs $G_1,G_2,...,G_n$is known as *cycle of graphs* and we shall denote it by $C(G_1,G_2,...,G_n)$. If we replace each vertices by a graph G i.e. $G_1 = G$, $G_2 = G$, . . ., $G_n= G$, such cycle of a graph G, we shall denote it by $C(n \cdot G)$.

Above definition 1.3 was introduced by Kaneria et. al. [4].

Definition−1.4:Let$G$beagraphand$G_1,G_2,...,G_n,n\geq 2$be$n$copiesofgraph$G$. Then the graph obtained by adding an edge from $G_i$to G_{i+1} (for $i = 1, 2, . . . , n − 1$)is called*pathunion*of$G$,weshalldenoteitby$P(G_1,G_2,...,G_n)$.Ifwereplaceeachgraphs G_1, G_2, . . . , $G_n$by a graph G i.e. $G_1 = G = G_2 = . . . = G_n$, such path union of $n$copies of G, we shall denote it by $P(n \cdot G)$.

For detail survey of various graph labelings and bibliographic references we refer to Gallian [2]. Labelled graph have many diversified applications. In [3] Somasunderam and Ponraj have introduced the notion of mean labeling of graphs in 2003. They proved that P_n, C_n, $P_n \times P_m$, $K_{2,m}$ are mean graphs and K_n, $K_{1,n}$ are mean graphs iff $n \leq 3$. They also prove that Wn is not a mean graph for n > 3.

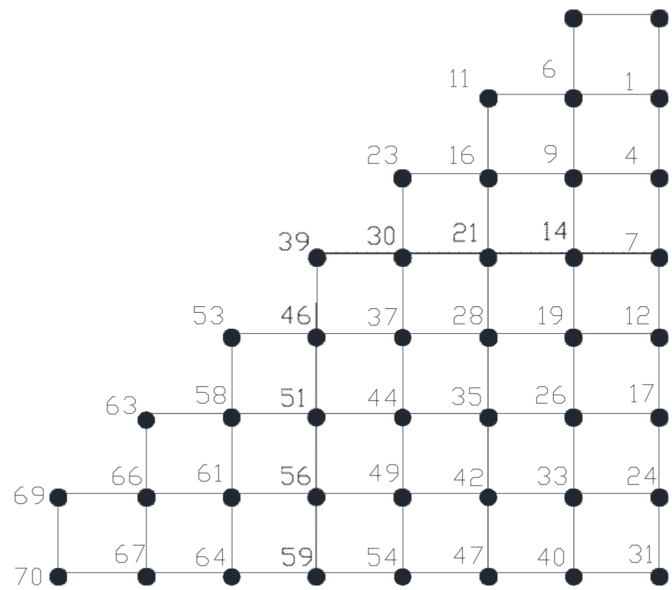
In [5] Kaneria et.al. prove that the step grid graph Stn where $n \geq 3$, is a mean graph with size n.

Definition1.5 Take$P_n,P_n,P_{n-1},\dots,P_2$pathson$n,n,n-1,n-2,\dots,3,2$verticesand arrange themvertically.

A graph obtained by joining horizontal vertices of given successive paths is known as a step grid graph of size n, where $n \geq 3$. It is denotedby St_n.

Obviously $|V(St_n)| = (n^2 + 3n - 2)$ and $|E(St_n)| = n^2 + n - 2$.

A Step grid graph St_8 with its mean labeling shown in *figure*−1.



Figure−1 Mean labeling of St_8.

They also proved that path union of step grid graph, cycle of step grid graph $C(r\cdot St_n)$ where $r \equiv 0$ (mod 2) are mean graphs.

A mean graph G will always have vertices with labels q, $q-1$ and 0, where $q \geq 2$.

Also two vertices with labels q and $q-1$ are adjacent in the mean graph G.

Inthispaperwehaveprovedthatpathunionofanymeangraphsisalsoameangraph and cycle of C_n, $P_n$and $P_n\times P_m$are mean graphs aswell.

2: MAIN RESULTS

Theorem−2.1: Path union of t copies of a mean graph G is also a meangraph.

Proof : Let G be a mean graph with injective mean labeling function $f : V(G) \longrightarrow$

$\{0, 1, \ldots, q\}$and bijective induced function $f^*: E(G) \longrightarrow \{1, 2, \ldots, q\}$.

Let $V(G) = \{v_i \,/\, i = 1, 2, \ldots, p\}$. Since $\exists v_i, v_j \in V(G)$ such that $f(v_i) = q$ and $f(v_j) = 0$, for some $i, j \in \{1, 2, \ldots, p\}$, without loss of generality we may assume that $f(v_1) = q$ and $f(v_p) = 0$.

Let H be the path union of t copies of the mean graph G. Let $u_{i,j}(1 \le j \le p)$ be vertices of i^{th} copy $G^{(i)}$ of path union H, $\forall i=1,2,\ldots,t$. Now join $u_{i,1}$ and $u_{i+1,p}$ by an edge when i is odd, join $u_{i,p}$ and $u_{i+1,1}$ by an edge when i is even, $\forall i=1,2,\ldots,t-1$ to form the graph H.

We define the labeling function $g: V(H) \longrightarrow \{0,1,\ldots,Q\}$, where $Q=t\cdot q+t-1$ as follows.

$g(u_{1,j})=f(v_j)+(Q-q)$, $\forall j= 1,2,\ldots,p$;

$g(u_{i,j}) = g(u_{i-1,j}) - (q+1)$, $\forall j=1,2,\ldots,p, \forall i=2,3,\ldots,t$.

Above labeling pattern give rise a mean labeling to the given H and so H is a mean graph.

Corollary−2.2: Path union of t copies of $K_{2,m}$ is a meangraph.

Proof : Let H be a path union of t copies of $K_{2,m}$. We see that the number of vertices in H is $|V(H)| = P = t(m + 2)$ and the number of edges in H is $E(H) = Q = 2tm + t - 1$.

Let $u_{i,1}, u_{i,2}, v_{i,j} (1 \le j \le m)$ be vertices of i^{th} copy $K_{2,m}^{(i)}$ of H, $\forall i=1,2,\ldots,t$. Now join $u_{i,1}$ and $u_{i+1,2}$ by an edge when i is odd, join $u_{i,2}$ and $u_{i+1,1}$ by an edge when i is even, $\forall\ i=1,2,\ldots,t-1$ to form path union of t copies of $K_{2,m}$.

We know that the labeling function $f: V(K_{2,m}^{(1)}) \longrightarrow \{0, 1, \ldots, q = 2m\}$ defined by

$f(u_{1,1})=q, f(u_{1,2})=0$ and

$f(u_{1,j}) = q - (2j - 1)$, $\forall j=1,2,\ldots,m$

is a mean labeling to the graph $K_{2,m}$. Now according to *Theorem*−2.1, we shall define $g: V(H) \longrightarrow \{0, 1, \ldots, Q\}$ as follows.

$g(u_{1,j})=f(u_{1,j})+(Q-q)$, $\forall j = 1,2$;

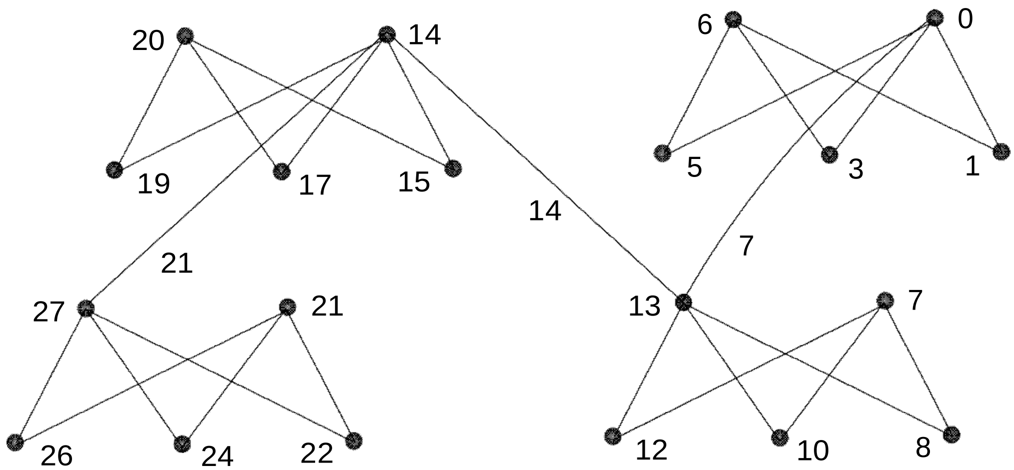
$g(v_{1,j})=f(v_{i,j})+(Q-q)$, $\forall j=1,2,\ldots,m$;

$g(u_{i,j}) = g(u_{i-1,j}) - (q+1)$, $\forall j=1,2, \forall i=2,3,\ldots,t$;

$g(v_{i,j}) = g(v_{i-1,j}) - (q+1)$, $\forall j=1,2,\ldots,m, \forall i=2,3,\ldots,t$.

Above labeling pattern give rise mean labeling to the path union of t copies of $K_{2,m}$ and so it is a mean graph.

***Illustration*−2.3:** Path union of 4 copies of $K_{2,3}$ and its mean labeling shown in *figure*−2.



Figure−2 Path union of 4 copies of $K_{2,3}$ and its mean labeling.

***Corollary*−2.4:** Path union of t copies of C_n is a mean graph.

Proof : Let H be path union of t copies of $C_n (n \in N)$. We see that number of vertices in H is $|V(H)| = P = tn$ and number of edges in H is $|E(H)| = Q = tn + t - 1$. Let $u_{i,j} (1 \leq j \leq n)$ be vertices of i^{th} copy $C^{(i)}_n$ of H, $\forall i = 1,2,...,t$. Now join $u_{i,1}$ and $u_{i+1,n}$ by an edge when i is odd, join $u_{i,n}$ and $u_{i+1,1}$ by an edge when i is even, $\forall i = 1,2,...,t-1$ to form path union of t copies of C_n.

We know that the labeling function $f : V(C_n^{(1)}) \longrightarrow \{0, 1, . . . , q = n\}$ defined by

$$f(u_{1,j}) = q + 1 - j, \quad \text{when } j \leq \left\lceil \frac{n+1}{2} \right\rceil$$

$$= q - j, \quad \text{when } j > \left\lceil \frac{n+1}{2} \right\rceil, \forall j = 1,2,...,n$$

is a mean labeling to the graph C_n. Now according to *Theorem*−2.1, we shall define

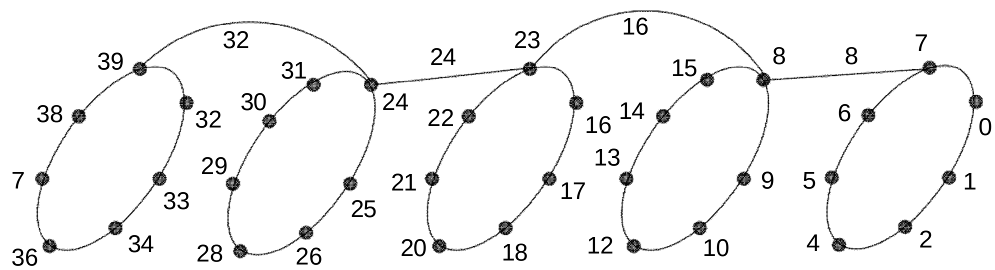
$g : V(H) \longrightarrow \{0, 1, . . . , Q\}$, as follows.

$$g(u_{1,j}) = f(u_{1,j}) + (Q - n), \quad \forall j = 1,2,...,n;$$

$$g(u_{i,j}) = g(u_{i-1,j}) - (n+1), \quad \forall j = 1,2,...,n, \forall i = 2,3,...,t.$$

Above labeling pattern give rise mean labeling to the graph H and so H is a mean graph.

Illustration−2.5:Pathunionof5copiesofC_7anditsmeanlabelingshownin*figure*−3.



Figure−3 Path union of 5 copies of C7 and its mean labeling.

Corollary−2.6: Path union of t copies of $P_n$is a meangraph.

Proof : Let H be a path union of t copies of $P_n(n \in N)$. We see that the number of verticesin$H$istnandthenumberofedgesin$H$is$tn-1$.Let$u_{i,j}(1 \le j \le n)$bevertices ofi^{th}copy$P^{(i)}$ of$H$, $\forall i=1,2,...,t$.Nowjoin$u_{i,1}$ and$u_{i+1,n}$byanedgewhen$i$isodd, join$u_{i,n}$and$u_{i+1,1}$ byanedgewhen$i$iseven,$\forall i=1,2,...,t-1$toformpathunionof t copies ofP_n.

We know that the labeling function $f: V(P_n^{(1)}) \longrightarrow \{0,1,...,q=n-1\}$definedby

$$f(u_{1,j}) = n - j, \qquad \forall j=1,2,...,n$$

is a mean labeling to the graph P_n. Now according to *Theorem*−2.1, we shall define

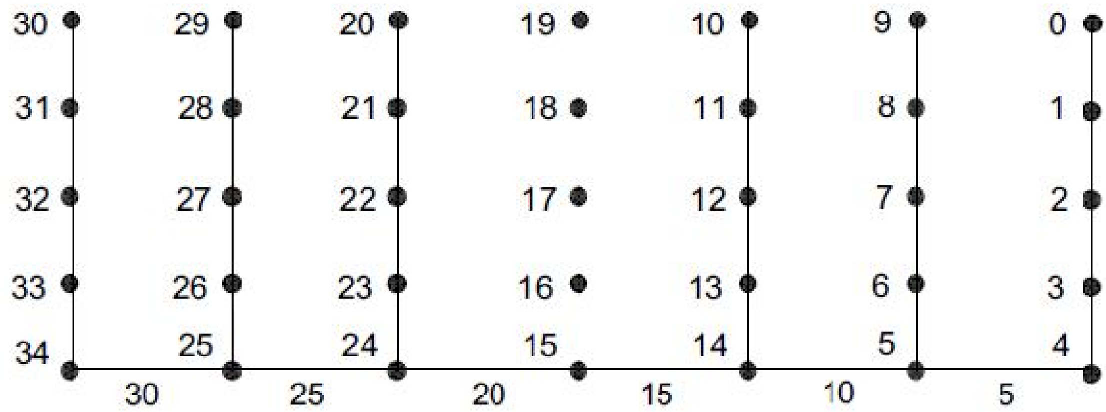
$g : V(H) \longrightarrow \{0, 1, . . . , Q\}$, where $Q = tn - 1$ as follows.

$$g(u_{1,j})=f(u_{1,j})+(Q-q), \qquad \forall j=1,2,...,n;$$

$$g(u_{i,j}) = g(u_{i-1,j}) - (q+1), \qquad \forall j=1,2,...,n, \forall i=2,3,...,t.$$

Above labeling pattern give rise mean labeling to the graph Hobtained by pathunion of$t$ copies of P_n and so it is a mean graph.

Illustration−2.7: Pathunion of 7 copies of P_5 and its mean labeling shown in *figure*−4.



Figure−4 Pathunion of 7 copies of P5 and its mean labeling.

Corollary−2.8: Path union of t copies of $P_n \times P_m$ is a meangraph.

Proof : Let H be a path union of t copies of $P_n \times P_m (m, n \in N - \{1\})$. We see that the number of vertices in H is $|V(H)| = P = tmn$ and the number of edges in H is $|E(H)| = Q = t(q + 1) - 1$, where $q = 2mn - (m + n)$. Let $u_{i,j,k} (1 \le j \le n,\ 1 \le k \le m)$ be vertices of i^{th} copy $(P_n \times P_m)^{(i)}$ of $H, \forall i = 1,2,...,t$. Now join $u_{i,n,m}$ with $u_{i+1,1,1}$ by an edge $\forall i=1,2,...,t-1$ to form the graph H.

We know that the labeling function $f : V((P_n \times P_m)^{(1)}) \longrightarrow \{0, 1, . . . , q\}$, where $q = 2mn - (m + n)$ defined by

$$f(u_{1,j,k}) = q - (2m-1)(j-1) - (k-1), \qquad \forall j=1,2,...,n, \forall k=1,2,...,m$$

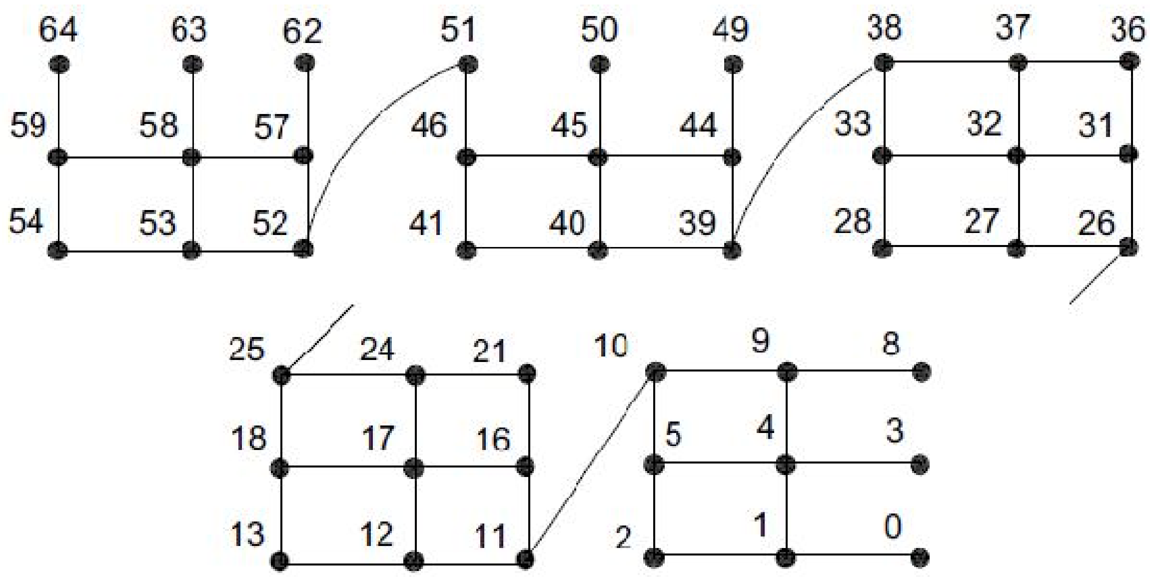
is a mean labeling to the graph $(P_n \times P_m)^{(1)}$. Now define $g : V(H) \longrightarrow \{0, 1, . . . , Q\}$ as follows.

$$g(u_{i,j,k}) = Q - (q + 1)(i - 1) - (2m - 1)(j - 1) - (k - 1),$$

$\forall i=1,2,...,t, \forall j=1,2,...,n, \forall k=1,2,...,m$.

Above labeling pattern give rise mean labeling to the graph H obtained by pathunion of t copies of grid graph $P_n \times P_m$ and so H is a mean graph.

Illustration−2.9 : Path union of 5 copies of $P_3 \times P_3$ and its mean labeling shown in *figure*−5.



Figure−5 Path union of 3 copies of P3 × P4 and its mean labeling.

Theorem−2.10: $C(t \cdot P_n)$ is a mean graph, where $t \equiv 0 \pmod 2$.

Proof : Let $G = C(t \cdot P_n)$, where $n \in N$. It is obvious that $P = |V(G)| = tn = Q = |E(G)|$. Let $u_{i,j} (1 \le j \le n, 1 \le i \le t)$ be vertices of graph G. We shall join $u_{i,1}$ with $u_{i+1,n}$, When $i + \frac{t}{2}$ is odd and $u_{i,n}$ with $u_{i+1,1}$, When $i + \frac{t}{2}$ is even to form the cycle graph $G = C(t \cdot P_n)$.

Now define the labeling function : $V(G) \longrightarrow \{0, 1, \ldots, Q\}$ as follows.

$g(u_{i,j}) = Q - n(i-1) - (j-1), \forall j = 1,2,\ldots,n, \forall i = 1,2,\ldots,\frac{t}{2};$

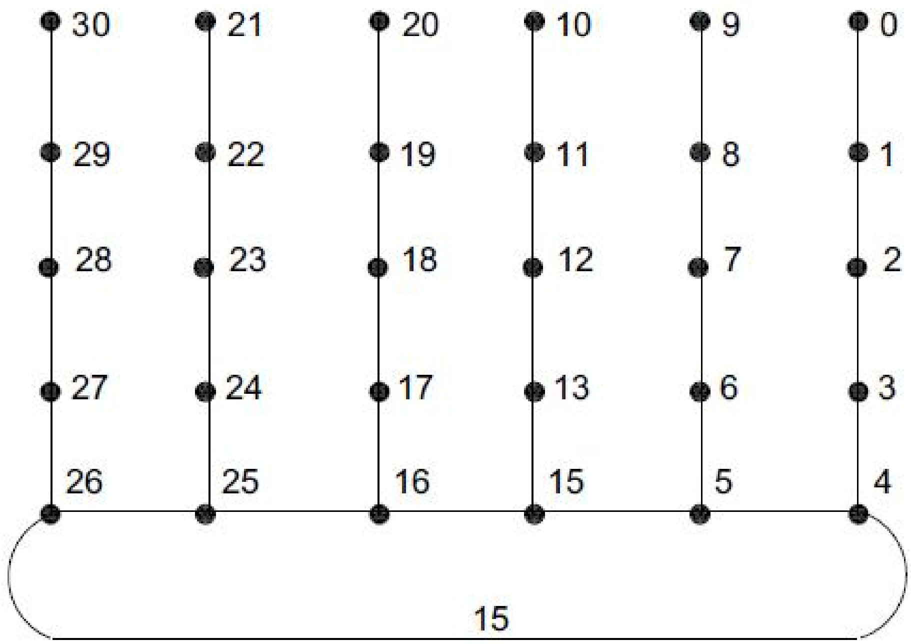
$g\left(u_{\frac{t}{2}+1,1}\right) = \frac{Q}{2};$

$g\left(u_{\frac{t}{2}+1,1}\right) = \frac{Q}{2} - j, \qquad \forall j = 1,2,\ldots,n;$

$g(u_{i,j}) = Q - n(i-1) - j, \forall j = 1,2,\ldots,n, \ \forall\, i = \frac{t}{2} + 2, \frac{t}{2} + 3, \cdots\cdots, t.$

Above labeling pattern give rise mean labeling to the graph G obtained by taking cycle of path P_n and so G is a mean graph.

Illustration−2.11 : $C(6 \cdot P_5)$ and its mean labeling shown in *figure*−6.



Figure−6 Cycle graph C(6 · P5) and its mean labeling.

Theorem−2.12: $C(t \cdot C_n)$ is a mean graph, where $n \in N$ and $t \equiv 0 \pmod 2$.

Proof : Let $G = C(t \cdot C_n)$, where $n \in N$. It is obvious that $P = |V(G)| = tn$ and $Q = |E(G)| = t(n + 1)$. Let $u_{i,j}(1 \le j \le n, 1 \le i \le t)$ be vertices of graph G. We shall join $u_{i,1}$ with $u_{i+1,n}$, when $i + \frac{i}{2}$ is odd and $u_{i,n}$ with $u_{i+1,1}$, when $i + \frac{i}{2}$ is even to form the cycle graph $G = C(t \cdot C_n)$.

Now define the labeling function : $V(G) \longrightarrow \{0, 1, \ldots, Q\}$ as follows.

$$g(u_{i,j}) = Q - (n + 1)(i - 1) - (j - 1), \quad \text{when } j \le \left\lceil \frac{n+1}{2} \right\rceil$$

$$= Q - (n+1)(i-1) - j, \quad \text{when } j > \left\lceil \frac{n+1}{2} \right\rceil,$$

$$\forall j = 1, 2, \ldots, n, \forall i = 1, 2, \ldots, \frac{t}{2};$$

$$g\left(u_{\frac{t}{2}+1,1}\right) = \frac{Q}{2};$$

$$g\left(u_{\frac{t}{2}+1,1}\right) = \frac{Q}{2} - j, \qquad \forall j = 1, 2, \ldots, \left\lceil \frac{n}{2} \right\rceil;$$

$= \frac{Q}{2} - (j + 1), \forall j = \left\lceil\frac{n+2}{2}\right\rceil, \left\lceil\frac{n+4}{2}\right\rceil, \ldots, n;$

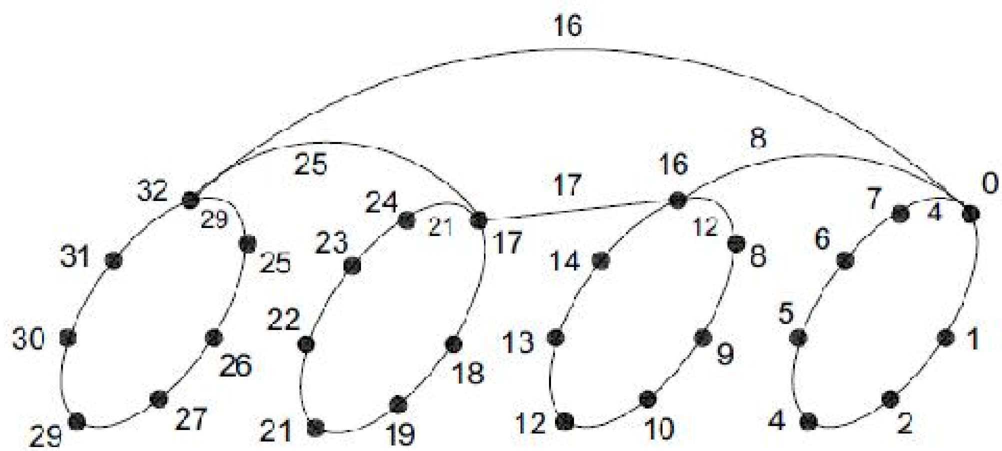
$g(u_{i,j}) = Q - (n + 1)(i - 1) - j$, when $j \leq \left\lceil\frac{n+1}{2}\right\rceil$

$= Q - (n + 1)(i - 1) - (j + 1)$, when $j > \left\lceil\frac{n+1}{2}\right\rceil$

$\forall j = 1,2,\ldots,n, \ \forall i = \frac{t}{2} + 2, \frac{t}{2} + 3, \cdots\cdots, t.$

Above labeling pattern give rise mean labeling to the cycle graph G obtained by C_n and so G is a mean graph.

Illustration *−2.13* **:** $C(4 \cdot C_7)$ and its mean labeling shown in *figure*−7.



Figure−7 Cycle graph C(4 · C7) and its mean labeling.

Theorem *−2.14:* $C(t \cdot P_n \times P_m)$ is a mean graph, where $m,n \in N$ and $t \equiv 0 (\text{mod} 2)$.

Proof: Let $G = C(t \cdot P_n \times P_m)$, where $n,m \in N$. It is obvious that $P = |V(G)| = tmn$ and $Q = |E(G)| = t(2mn - (m + n) + 1)$. Let $u_{i,j,k}(1 \leq j \leq n, 1 \leq k \leq m, 1 \leq i \leq t)$ be vertices of graph G. We shall join $u_{i,1,1}$ with $u_{i+1,n,m}, \forall i = 1,2,\ldots,t-1$ to form the cycle graph $G = C(t \cdot P_n \times P_m)$.

Now define the labeling function: $V(G) \longrightarrow \{0,1,\ldots,Q\}$, where $Q = t(q+1)$ and $q = 2mn - (m + n)$ as follows.

$g(u_{i,j,k}) = Q - (q + 1)(i - 1) - (2m - 1)(j - 1) - (k - 1),$

$\forall j = 1, 2, \ldots, n, \forall k = 1, 2, \ldots, m, \forall i = 1, 2, \ldots, \frac{t}{2};$

$g\left(u_{\frac{t}{2}+1,1,1}\right) = \frac{Q}{2}$;

$$g\left(u_{\frac{t}{2}+1,j,k}\right) = Q - (q+1)(i-1) - (2m-1)(j-1) - k,$$

$\forall j = 1, 2, \ldots, n, \forall k = 1, 2, \ldots, m,$

$$g(u_{i,j}) = Q - (n+1)(i-1) - j,$$

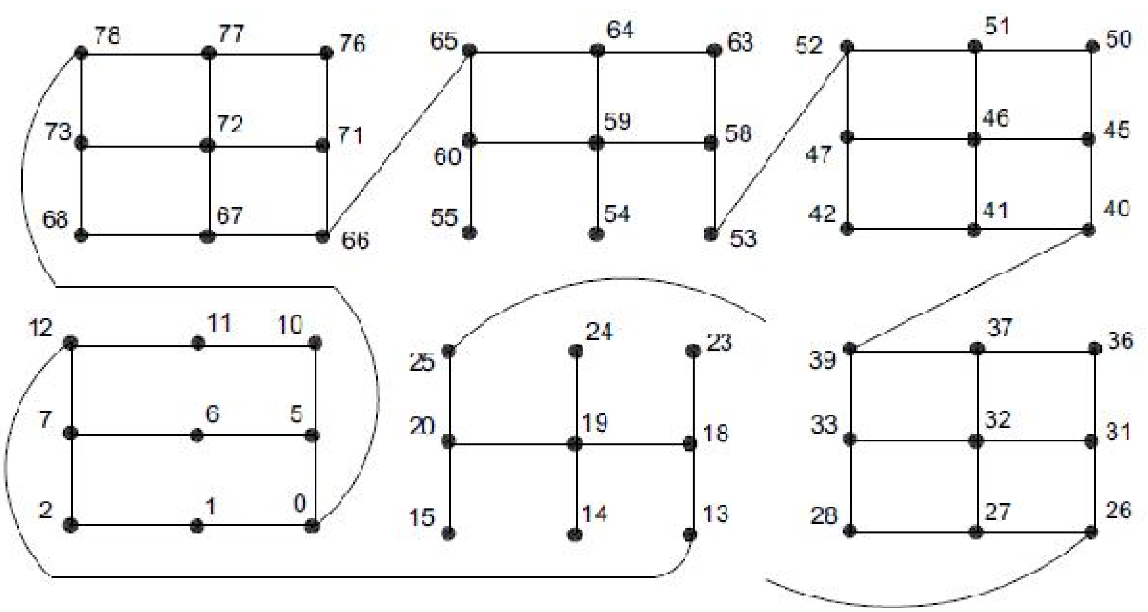
$\forall j = 1, 2, \ldots, n, \forall k = 1, 2, \ldots, m;$

$g(u_{i,j,k}) = Q - (q+1)(i-1) - (2m-1)(j-1) - k,$

$\forall j = 1, 2, \ldots, n, \forall k = 1, 2, \ldots, m, \forall i = \frac{t}{2} + 2, \frac{t}{2} + 3, \ldots, t.$

Above labeling pattern give rise mean labeling to the graph G obtained by taking cycle of grid graph $(P_n \times P_m)$ and so G is a mean graph.

***Illustration*−2.15 :** $C(6 \cdot P_3 \times P_3)$ and its mean labeling shown in *figure*−8.



Figure−8 Cycle graph C(6 · P3 × P3) and its mean labeling.

3: CONCLUDING REMARKS

Here we have discussed mean labeling for path union of C_n, P_n, $K_{2,m}$and $P_n\times P_m$. Also we proved that cycle of C_n, P_n, $P_n\times P_m$are mean graphs. These results contribute somenewtopicstothefamiliesofmeangraphs.Thelabelingpatternisdemonstratedby means of illustrations.

Theorem−2.1 is a strong result of general nature, as it shows $P(t_1\cdot P(t_2\cdot K_{2,m}))$, $P(t_1\cdot P(t_2\cdot P_n))$,$P(t_1\cdot P(t_2\cdot C_n))$,$P(t_1\cdot P(t_2\cdot (P_n\times P_m)))$,$P(t_1\cdot C(t_2\cdot P_n))$,$P(t_1\cdot C(t_2\cdot C_n))$ and$P(t_1\cdot C(t_2\cdot (P_n\times P_m)))$aremeangraphs.Weraiseopenquestiontogetmeanlabeling for the graphs $C(t_1\cdot C(t_2\cdot P_n))$,$C(t_1\cdot C(t_2\cdot C_n))$,$C(t_1\cdot C(t_2\cdot (P_n\times P_m)))$.

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