Research Article **Available online www.ijsrr.org** **ISSN: 2279–0543**

International Journal of Scientific Research and Reviews

Deferred Statistical Boundedness

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ABSTRACT

In this paper, the concept of deferred statistical boundedness is introduced and it is investigated that deferred statistical boundedness is a natural gener alization of boundedness.

KEYWORDS: boundedness, *monotone, algebra, symmetric*

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1 INTRODUCTION

The idea of statistical convergence which is, in fact, a generalization of the usual notion of convergence was introduced by Fast[1] and Steinhaus[2] independently in the same year 1951 and since then several generalizations and applications of this concept have been investigated by various authors namely Bhardwaj *et al.*[3,4,5] Connor[6], Et[7], Fridy[8], Mursaleen[9], Rath and Tripathy[10], Salat[11] and many others.

The natural density $\delta(A)$ of a subset A of the set N of natural numbers is defined as

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n} \, |\{k \le n : k \in A \}|$$

Where $|\{k \le n : k \in A \}|$ denotes the number of elements of A not exceeding n. Obviously, we have $\delta(A) = 0$, provided that A is a finite set.

A sequence $x = (x_k)$ is said to be statistically convergent to L if for every $\varepsilon > 0$,

$$\delta(\{k \in N : |x_k - L| > \varepsilon\}) = 0,$$

$$\text{i.e., } \lim_{n\to\infty} \frac{1}{n} \, |\{k \le n : |x_k - L| > \varepsilon \}| = 0.$$

In this case we write $St\text{-}\lim x_k = L$. Since $\lim x_k = L$ implies $St\text{-}\lim x_k = L$, statistical convergence may be considered as a regular summability method. The set of all statistically convergent sequences is denoted by S^t.

R. P Agnew[12] introduced the concept of deferred Ceşaro mean of real (or complex) valued sequences (x_k) defined by

$$(D_{p,q}\, x)_n = \frac{1}{p(n)-q(n)} \sum_{k=p(n)+1}^{q(n)} x_k\,, \quad n = 1, 2, 3, \ldots$$

where $p = \{ p(n) : n \in N \}$ and $q = \{ q(n) : n \in N \}$ are the sequences of non- negative integers satisfying $p(n) < q(n)$ satisfying $\lim_{n\to\infty} q_n = \infty$.

The concepts of deferred density and deferred statistical convergence were given by Küçükaslan and Yılmaztürk[13,14] as follows :

Let A be a subset of N and denote the set $\{k : p(n) < k \le q(n),\, k \in A\}$ by $A_{p,q}(n)$. The deferred density of A is defined by

$$\delta_{p,q}(A) = \lim_{n\to\infty} \frac{1}{q(n)-p(n)} \left|A_{p,q}(n)\right|, \text{ provided the limit exists.}$$

The vertical bars indicate the cardinality of the enclosed set $A_{p,q}(n)$. If $q(n) = n$, $p(n) = 0$, then deferred density coincides with natural density of A.

A real valued sequence $x = (x_k)$ is said to be deferred statistically convergent to L,

if for each $\varepsilon > 0$

$$\lim_{n \to \infty} \frac{1}{q(n) - p(n)} |\{p(n) < k \leq q(n) : |x_k - L| > \varepsilon \}| = 0.$$

In this case we write $S_{p,q} - \lim x_k = L$. The set of all deferred statistically convergent sequences will be denoted by $S_{p,q}$. If $q(n) = n$, $p(n) = 0$, then deferred statistical convergence coincides with usual statistical convergence.

The concept of statistical boundedness was given by Fridy and Orhan[15] as follows:

The real number sequence x is statistically bounded if there exists a number

$B \geq 0$ such that $\delta(\{k : |x_k| > B\}) = 0$.

It can be shown that every bounded sequence is statistically bounded, but the converse is not true. For this consider a sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} k, & \text{if } k \text{ is a square} \\ 1, & \text{if } k \text{ is not a square.} \end{cases}$$

Clearly $x = (x_k)$ is not a bounded sequence, but it is statistically bounded.

The set of all statistically bounded sequences is denoted by $S(b)$.

Before proceeding further, we recall some definitions which will be needed in the sequel.

A sequence space X is called

- *monotone* if it contains the canonical preimages of all its stepspaces,
- *symmetric*, if $(x_k) \in X$ implies $(x_{\pi(k)}) \in X$, where π is a permutation of N,
- *sequence algebra* if $x.y \in X$, whenever $x, y \in X$
- *solid* (or *normal*), if $(\alpha_k x_k) \in X$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$ for all k $\in$ N and $(x_k) \in X$.

In the present paper, the concept of deferred statistical boundedness is introduced and the relation between statistical boundedness and deferred statistical boundedness is established.

Throughout the paper, we consider the sequences of non negative integers p ={ p(n) : n $\in N$ } and q ={ q(n) : n $\in N$ } are the sequences of non- negative integers satisfying p(n) < q(n) satisfying $\lim_{n\to\infty} q_n = \infty$.

Any other restriction (if needed) on $p(n)$ and $q(n)$ will be mentioned in the related theorems.

2 DEFERRED STATISTICAL BOUNDEDNESS

Definition 2.1 *The sequence* $x = (x_k)$ *is said to be deferred statistically bounded (* $S_{p,q}$*–bounded sequence) if there exists* $M \geq 0$ *such that*

$$\lim_{n\to\infty} \frac{1}{q(n)-p(n)} |\{p(n) < k \leq q(n) : |x_k| > M \}| = 0,$$

i.e., $\delta_{p,q}(\{k : |x_k| > M\}) = 0.$

The set of all deferred statistically bounded sequences of order will be denoted by $S_{p,q}(b)$*. For* $q(n) = n$*,* $p(n) = 0$ *we shall write* $S(b)$ *instead of* $S_{p,q}(b)$.

Theorem 2.2 Every *bounded sequence is deferred statistically bounded, but the con verse is not true.*

Proof. Bounded sequences are obviously deferred statistically bounded as the empty set has zero deferred density. However, the converse is not true, as the following example demonstrates.

Consider a sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} k, & if\ k\ is\ a\ square \\ 1, & if\ k\ is\ not\ a\ square. \end{cases}$$

Clearly $x = (x_k)$ is not a bounded sequence. However, for $q(n) = n$,

$p(n) = 0$ we have $x = (x_k) \in S_{p,q}(b)$.

Theorem 2.3 *Every deferred statistically convergent sequence is deferred statistically bounded, but the converse is not true.*

Proof. Let $x \in S_{p,q}$ and $\varepsilon > 0$ be given. Then there exists L such that

$$\lim_{n\to\infty} \frac{1}{q(n)-p(n)} |\{p(n) < k \leq q(n) : |x_k - L| > \varepsilon \}| = 0.$$

The result follows from the inequality

$$|\{p(n) < k \leq q(n) : |x_k| \geq |L| + \varepsilon \}| \leq |\{p(n) < k \leq q(n) : |x_k - L| \geq \varepsilon \}|$$

To show the strictness of the inclusion, choose $q(n) = n$, $p(n) = 0$ and consider the sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} 1, \; if \; k = 2n \\ -1, if \; k = 2n+1. \end{cases}$$

Then $x \in S_{p,q}$ (b) but $x \notin S_{p,q}$.

Remark 2.4 *It can be shown that not every sequence is deferred statistically bounded. For this; let* $q(n) = n$, $p(n) = 0$ *and consider a sequence defined by* $x = (x_k) = (1, 2, 3, , ...)$. *Then for any* $L \geq 0$, *we have* $\{k : |x_k| > L\} = \mathrm{N} - S$, *where S is a finite subset of* N *and so* $\delta(\{k : |x_k| > L\}) = 1$, *thus* $x = (x_k)$ *is not statistically bounded*

Theorem 2.5 *Every convergent sequence is deferred statistically bounded , but the converse is not true.*

Proof. Since every convergent sequence is deferred statistically convergent, so the result follows in view of the Theorem 2.3. For the converse part consider a sequence $x = (x_k)$ defined as

$$x_k = \begin{cases} 2, \; if \; k \; is \; a \; perfect \; square \\ 0, if \; kis \; not \; a \; perfect \; square. \end{cases}$$

Then we have

$$\frac{1}{q(n) - p(n)} |\{p(n) < k \leq q(n) : |x_k - 0| \geq \epsilon\}| \leq \frac{\sqrt{q(n)} - \sqrt{p(n)} + 1}{q(n) - p(n)}$$

and so $x = (x_k)$ is deferred statistically convergent with $S_{p,q} - \lim x_k = 0$, so x is deferred statistically bounded by Theorem 2.3, but it is not convergent.

Theorem 2.6 *(i)* $S_{p,q}(b)$ *is not symmetric.*

(ii) $S_{p,q}(b)$ *is normal and hence monotone.*

(iii) $S_{p,q}(b)$ *is a sequence algebra.*

Proof. (*i*) Let $x = (x_k) = (1, 0, 0, 2, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 4, ...) \in S_{p,q}(b)$. Let $y = (y_k)$ be an arrangement of (x_k) which is defined as follows:

$(y_k) = (x_1, x_2, x_4, x_3, x_9, x_5, x_{16}, x_6, x_{25}, x_7, x_{36}, x_8, x_{49}, x_{10,\ldots}) = (1, 0, 2, 0, 3, 0, 4, 0, 5, 0, \ldots)$.

Clearly for any M > 0, $\delta_{p,q}$ ({k : $|y_k|$ > M}) = 0, for p(n) = n, q(n)= 0.

(ii) Let $x = (x_k) \in S_{p,q}(b)$ and $y = (y_k)$ be a sequence such that $|y_k| \le |x_k|$ for all $k \in$ N. Since $x \in S_{p,q}$ (b) there exists a number M such that $\delta_{p,q}(\{k : |x_k| > M\}) = 0$.

Clearly $y \in S_{p,q}(b)$ as $\{k : |y_k| > M\} \subset \{k : |x_k| > M\}$. So $S_{p,q}(b)$ is normal. It is well known that every normal space is monotone, so $S_{p,q}(b)$ is monotone.

(iii) Let $x, y \in S_{p,q}(b)$. Then there exists $K, M > 0$ such that $\delta_{p,q}(\{k : |x_k| \ge K\}) = 0$ and $\delta_{p,q}(\{k : |y_k| \ge M\}) = 0$. The proof follows from the following inclusion $\{k : |x_k y_k| \ge KM\} \subset \{k : |x_k| > K\} \cup \{k : |y_k| > M\}$.

ACKNOWLEDGEMENT

The author expresses his sincere thanks to the referees for their valuable suggestions and comments, which improved the presentation of this paper**.**

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