Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

On Commutativity of Non-Associative Primitive Rings with $(xy)^2 - xy$ ∈Z(R)

Y.Madana Mohana Reddy [1]* and G.Shobha Latha[2]

[1]Department of Mathematics, Rayalaseema University, Kurnool –518007, AP. India
[2]Department of Mathematics, S.K.Univesity, Ananthapuramu – 515003,AP.India.

ABSTRACT:

It is well known that a Boolean ring satisfies$x^2 = x$, for all x∈R and this implies commutativity. Similarly we can see the properties of rings in which$(xy)^2 = xy$ for each pair of elements x, y ∈ R. In **Quadri**and others proved that an associative semi prime ring in which$(xy)^2 - xy \in Z(R)$ is commutative. In this direction we prove that a 2-torsion free non-associative ring with unity satisfying $(xy)^2 - xy \in Z(R)$ or $(xy)^2 - yx \in Z(R)$is commutative. We give an example to show that the unity is essential in the hypothesis. Also, we prove that a non-associative primitive ring (not necessarily having unity) satisfying $(xy)^2 - xy$ (or) $(xy)^2 - yx$is central for all x, y ∈ R is commutative.

KEYWORDS: Center, Torsion Free Ring, Primitive Ring.

*Corresponding author

Dr. Y.Madana Mohana Reddy,

Department of Mathematics,

Rayalaseema University,

Kurnool – 518007, A.P., India.,

Email:madanamohanareddy5@gmail.com

Mob. No: 9491377084

INTRODUCTION:

The study of associative and non- associative rings has evoked great interest and assumed importance. The results on associative and non- associative rings in which one does assume some identities in the center have been scattered throughout the literature.

Many sufficient conditions are well known under which a given ring becomes commutative. Notable among them are some given by Jacobson, Kaplansky and Herstein. Many Mathematicians of recent years studied commutativity of certain rings with keen interest. Among these mathematicians Herstein, Bell, Johnsen, Outcalt, Yaqub, Quadri and Abu-khuzam are the ones whose contributions to this field are outstanding.

PRELIMINARIES:

Non – Associative Ring:

If R is an abelian group with respect to addition and with respect to multiplication R is distributive over addition on the left as well as on the right.

For every elements x, y, z of R

(x+y)z = xz+yz,z(x+y) = zx+zy

Alternative rings, Lie rings and Torsian rings are best examples of these non–associative rings.

Commutator:

For every x, y in a ring R satisfying $[x, y] = xy - yx$ then $[x, y]$ is called a commutator

Commutative Ring:

For every x, y in a ring R if $xy = yx$ then R is called a commutative ring.

Non–commutative ring is split from the commutative ring, i.e., R is not commutative with respect to multiplication. i.e., we cannot take $xy = yx$ for every x, y in R as an axiom.

Semi Prime Ring:

A ring R is semi prime if for any ideal A of R, $A^2 = 0$ implies A = 0. These rings are also referred to as rings free from trivial ideals.

Primitive Ring:

A ring R is defined as primitive in case it possesses a regular maximal right ideal, which contains no two–sided ideal of the ring other than the zero ideal.

Torsion–Free ring:

If R is m-torsion free ring, then mx=0 implies x=0 for positive integer m and x is in R.

Center :

In a ring R, the center denoted by Z(R) is the set of all elements $x \in R$ such that xy=yx for all $x \in R$. It is important to note that this definition does not depend on the associative of multiplication and in fact, we shall have occasion to deal with derivation of non-associative algebras.

MAIN RESULTS:

Theorem 1**:** Let R be a 4-torsion free non-associative ring with unity satisfying.

(i) $[(xy)^2 - xy, y] = 0$or (ii) $[(xy)^2 - xy, x] = 0$, Then R is commutative.

Proof: Hypothesis (i) yields$[(xy)^2 - xy]y = y[(xy)^2 - xy]$ ..1.1

Putting x+1 for x in 1.1 and using 1.1, we get [(xy)y]y+[y(xy)]y=y[y(xy)]+y[(xy)y] ..1.2

Now replacing y by y+1in 1.2 and simplifying it by keeping the brackets unchanged, we obtain[(xy)y]+[y(xy)]y+4(xy)y+y(xy)+(yx)y+5xy+yx+2x

=y[y(xy)]+y[y(xy)]+(xy)y+4y(xy)+3xy+3yx+y(yx)+2x ..1.3

Now using 1.2 in equation 1.3, we get

3(xy)y+(yx)y+2xy=3y(xy)+y(yx)+2yx, ..1.4

Expanding 1.4 by replacing y by y+1, then substituting 1.4 in this expansion, we get 7xy+yx=5yx+3xy.

i.e., 4(xy-yx) =0.

But R is a 4-torsion free. Hence xy–yx =0,

i.e., xy=yx. Thus R is commutative.

Similarly, R is commutative if R satisfies (ii).

Theorem 2**:** Let R be a 2-torsion free non-associative ring with unity satisfying

$(xy)^2 - xy \in Z(R)$ for all x,y in R. Then R is commutative.

Proof: By hypothesis $(xy)^2 - xy \in Z(R)$.. ...2.1

Replacing x by x+1 in 2.1, and using 2.1 we get

$(xy)y + y(xy) + y^2 - y \in Z(R)$ ...2.2

Again replacing x by x+1 in 2.2 and using it, we obtain $2y^2 \in Z(R)$..

Since R is a 2-torsion free.$y^2 \in Z(R)$.. ...2.3

Replacing y by xy in 2.3, we get $(xy)^2 \in Z(R)$ 2.4

But by hypothesis

$(xy)^2 - xy \in Z(R)$, hence we get $xy \in Z(R)$. ...2.5

Now again replacing x by x+1 in 2.5,

we get xy+yx∈z(R). ...2.6

From the equations 2.5and 2.6 we obtain y∈Z(R) for all y∈R.

Hence R is commutative.

REFERENCES:

1. Bell H.E. On some commuativity theorems of Herstein Arch. Math., 1973; 24: 34-38.
2. AbuKhuazam. H, A Commutativity Theorem for Rings, Math. Japonica., 1980; 25: 593-595.
3. Girl. R.D.Rakhunde, R.R., Dhoble et al A.R. On commutativity of non- associative rings, The Math. Stu., 1992; 61(1-4): 149-152.