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Dominator Chromatic Number of Grid Graphs

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ABSTRACT

Let G be a graph with minimum degree at least one. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number of G is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G . In this paper, we obtain total dominator chromatic number of grid graphs.

Mathematics Subject Classification : 05C15, 05C69

KEY WORDS : Total dominator chromatic number, ladder graph, grid graph.

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INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let $G=(V, E)$ be a graph of order n with minimum degree atleast one . The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $N[v]=N(v) \cup \{v\}$. The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H , we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1=u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1=v_2$. A grid graphs can be defined as $P_m \times P_n$ where $m, n \geq 2$. A ladder graph can be defined as $P_2 \times P_n$ where $n \geq 2$ and is denoted by L_n . A subset S of V is called a total dominating set if every vertex in V is adjacent to some vertex in S . The total dominating set is minimal total dominating set if no proper subset of S is a total dominating set of G . The total domination number γ_t is the minimum cardinality taken over all minimal total dominating set of G . A γ_t -set is any minimal total dominating set with cardinality γ_t .

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A total dominator coloring (td-coloring) of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G . This concept was introduced by A.Vijiyalekshmi in[2]. This notion is also referred as a smarandachely k -dominator coloring of G ($k \geq 1$) and was introduced by A.Vijiyalekshmi in[4]. For an integer $k \geq 1$, a smarandachely k -dominator coloring of G is a proper coloring of G such that every vertex in G properly dominates a k color class. The smallest number of colors for which there exist a smarandachely k -dominator coloring of G is called the smarandachely k -dominator chromatic number of G , and is denoted by $\chi_{td}^s(G)$.

In a proper coloring C of G , a color class of C is a set consisting of all those vertices assigned the same color. Let C^l be a minimal td-coloring of G . We say that a color class $c_i \in C^l$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G . These color classes are also called repeated color classes.

The total dominator chromatic number of paths, cycles and ladder graphs were found in [3].

We have the following observations from [3].

Theorem A [3] Let G be p_n or C_n . Then

$$\chi_{td}(p_n) = \chi_{td}(C_n) = \begin{cases} 2 \left\lfloor \frac{n}{4} \right\rfloor + 2 & \text{if } n \equiv 0 \pmod{4} \\ 2 \left\lfloor \frac{n}{4} \right\rfloor + 3 & \text{if } n \equiv 1 \pmod{4} \\ 2 \left\lfloor \frac{n+2}{4} \right\rfloor + 2 & \text{otherwise} \end{cases}$$

Theorem B [3] For every $n \geq 2$, the total dominator chromatic number of a ladder graph L_n is

$$\chi_{td}(L_n) = \begin{cases} 2 \left\lfloor \frac{p}{6} \right\rfloor + 2 & \text{if } p \equiv 0 \pmod{6} \\ \begin{cases} 2 \left\lfloor \frac{p-2}{6} \right\rfloor + 4 \\ 2 \left\lfloor \frac{p-4}{6} \right\rfloor + 4 \end{cases} & \text{otherwise} \end{cases}$$

In this paper, we obtain the least value for total dominator chromatic number for grid graphs.

Main Results

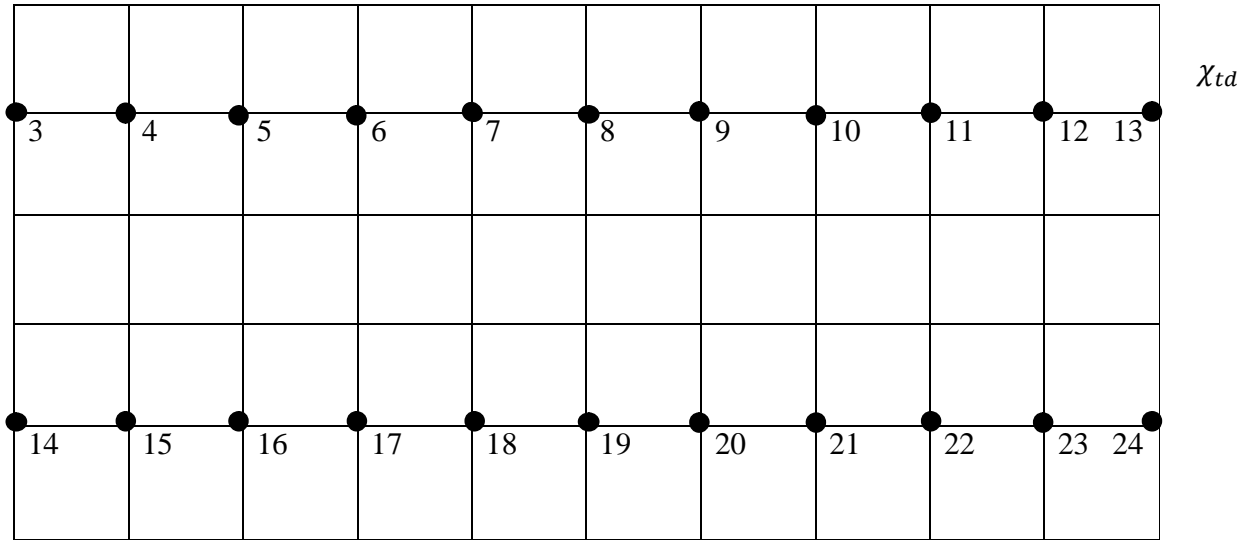
Notations: We denote $G_{m,n} = P_m \times P_n$ and let $V(G_{m,n}) = \{u_{ij} / 1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$.

Theorem 1 $\chi_{td}(G_{m,n}) = \frac{mn}{3} + 2$ if either m or $n \equiv 0 \pmod{3}$.

Proof: Let $m=3p$, $p \in \mathbb{Z}^+$. Proof is by using induction on p . For $1 \leq i \leq p$, let $D_{i,n} = \{u_{(i+1,j)} / \substack{1 \leq j \leq n \\ 1 \leq i \leq k+1}\}$ be a γ_t -set of $G_{3,n}$. We assign n distinct colors say $3, 4, 5, \dots, (n+2)$ to all vertices of $D_{i,n}$. Also we assign two repeated colors say $1, 2$ to the vertices u_{ij} and $u_{kl} \in V(G_{3,n}) - D_{i,n}$ such that $|i - k| + |j - l| = 1$. So $\chi_{td}(G_{3,n}) = n+2 = \frac{mn}{3} + 2$. By induction hypothesis, we assume that the theorem is true for $p=k$ and so $\chi_{td}(G_{3k,n}) = kn+2 = \frac{mn}{3} + 2$. For $p=k+1$, first for td-coloring of $G_{3k,n}$, we need $kn+2$ colours, by induction hypothesis. Since in a td-coloring of $G_{3(k+1),n}$, we can already use repeated colors 1 and 2 in the vertices $V(G_{3k,n}) - D_{i,n}$ followed by $G_{3(k+1),n}$ as earlier and we assign $n(k+1)$ different colors to the vertices of $D_{i,n}$ for $1 \leq i \leq k+1$. So $\chi_{td}(G_{3(k+1),n}) = n(k+1)+2 = \frac{mn}{3} + 2$.

Illustration:

Consider $G_{6,11}$



$(G_{6,11})=24$

Fig.1

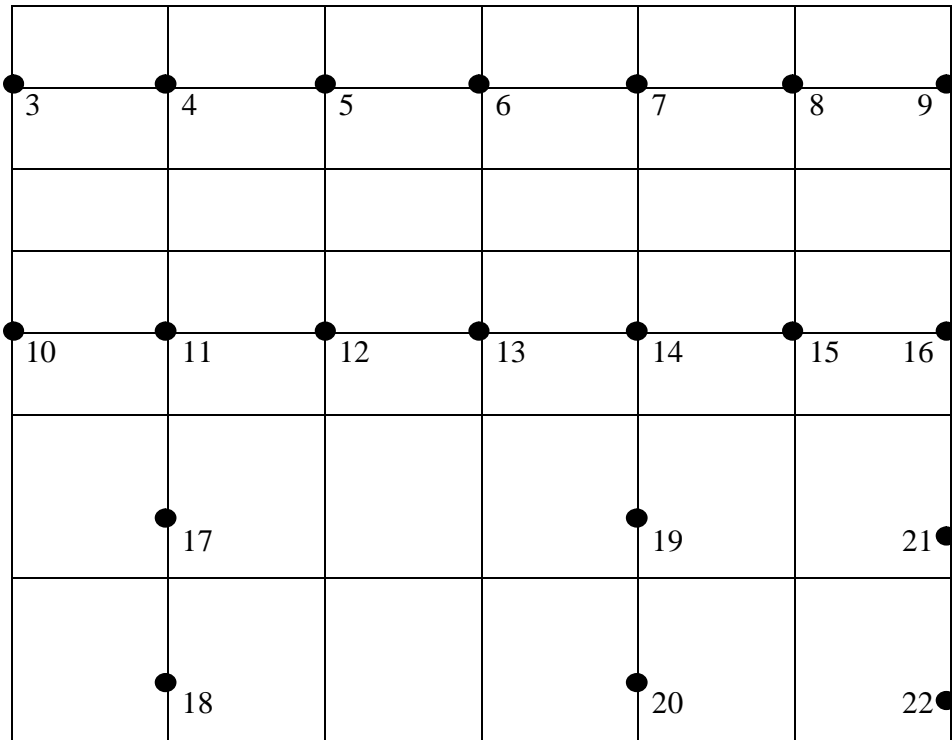
Theorem 2 $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m-2,n}) + \chi_{td}(L_n) - 2$ if $m \equiv 2(mod 3)$ and $n \equiv 1,2(mod 3)$.

Proof: We have $G_{m,n}$ is obtained by $G_{m-2,n}$ followed by $G_{2,n}$. Since in a td-coloring of $G_{m,n}$, we cannot use the non-repeated colors of vertices in $G_{m-2,n}$, for the $G_{2,n}$ and we can use the same repeated colors of vertices in the graphs $G_{m-2,n}$ and $G_{2,n}$. Since $m - 2 \equiv 0(mod 3)$ and

$$\chi_{td}(G_{m-2,n}) = \frac{(m-2)n}{3} + 2. \text{ Thus } \chi_{td}(G_{m,n}) = \chi_{td}(G_{m-2,n}) + \chi_{td}(L_n) - 2. \square$$

Illustration:

Consider $G_{8,7}$



$\chi_{td}(G_{8,7})=22$

Fig.2

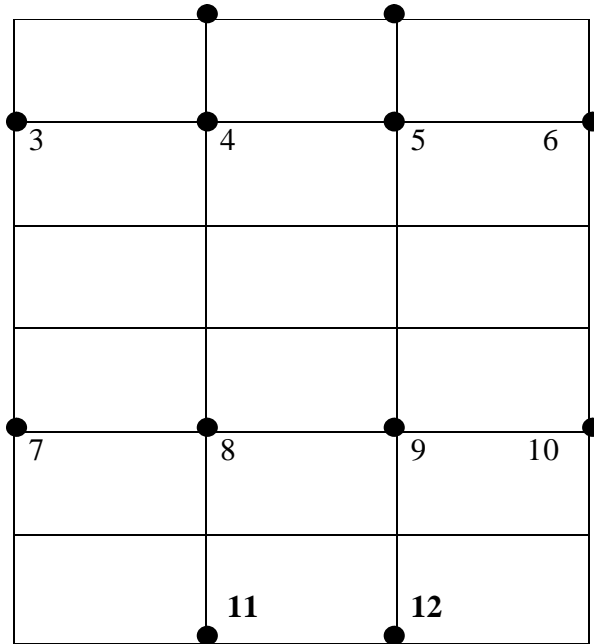
Theorem 3 For $m, n \equiv 1 \pmod{3}$,

$$\chi_{td}(G_{m,n}) = \begin{cases} \chi_{td}(G_{m,n-1}) + \chi_{td}(P_m) - 2 & \text{if } m \leq n \\ \chi_{td}(G_{m-1,n}) + \chi_{td}(P_n) - 2 & \text{if } m \geq n \end{cases}$$

Proof: Let $m, n \equiv 1 \pmod{3}$, so $(m-1), (n-1) \equiv 0 \pmod{3}$. Let $D_{m,n-1}$ be the γ_t set of $G_{m,n-1}$ and $|D_{m,n-1}| = \frac{m(n-1)}{3}$. Suppose that $|V(G_{m,n}) \cap D_{m,n-1}| = \frac{m(n-1)}{3}$ holds for $\frac{m(n-1)}{3}$ - layer P_{n-1} . We now assign $\frac{m(n-1)}{3}$ distinct colors to the vertices of $D_{m,n-1}$ and two repeated colors say 1 and 2 to the remaining vertices such that adjacent vertices receive different colors. Since the graph $G_{m,n}$ is $G_{m,n-1}$ followed by P_m , $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-1}) + \chi_{td}(P_m)$. Also the already used repeated colors are used in the coloring of P_m . So $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-1}) + \chi_{td}(P_m) - 2$. Proof is similar for the case $m \geq n$. \square

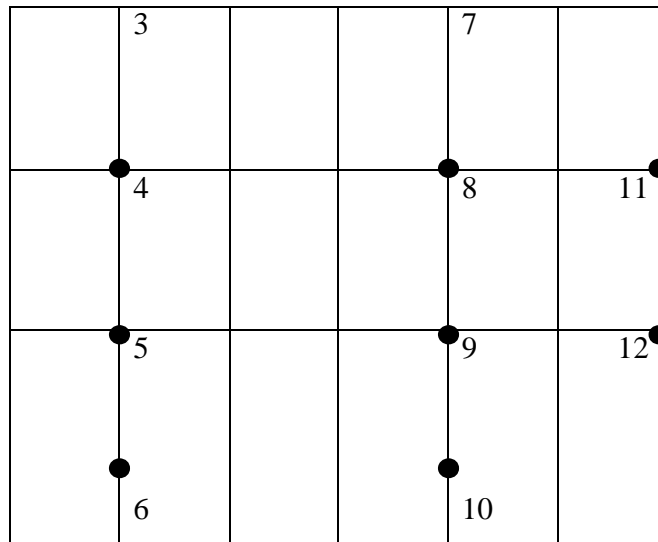
Illustration:

Consider $G_{4,7}$ and $G_{7,4}$



$\chi_{td}(G_{4,7})=12$

Fig.3



$\chi_{td}(G_{7,4})=12$

Fig.4

Theorem 4

$$\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-2}) + \chi_{td}(L_m) - 2 \text{ if } m \equiv 1(\text{mod } 3) \text{ and } n \equiv 2(\text{mod } 3).$$

Proof: Since $n - 2 \equiv 0(\text{mod } 3)$, $\chi_{td}(G_{m,n-2}) = \frac{m(n-2)}{3} + 2$. $G_{m,n}$ is got from $G_{m,n-2}$ followed by L_m .

From theorem 2, $\chi_{td}(G_{m,n}) = \chi_{td}(G_{m,n-2}) + \chi_{td}(L_m) - 2$. □

Illustration:

Consider $G_{7,8}$

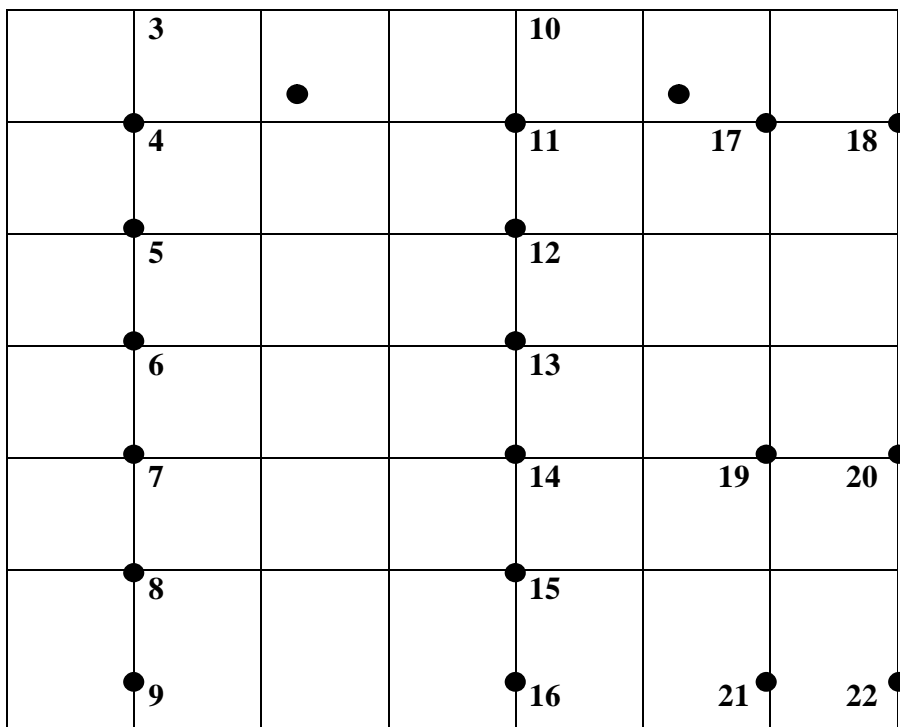


Fig .5

$$\chi_{td}(G_{7,8}) = 22$$

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