Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Total Dominator Chromatic Number on Various Classes of Graphs

Dr.A.Vijayalekshmi*[*1] and S.Anusha[2]

[1]Dept. of Mathematics, S.T.Hindu College, Nagercoil-629002,
Tamilnadu, India. **Email:**vijimath.a@gmail.com
[2] Dept. of Mathematics (S.S), S.T.Hindu College, Nagercoil-629002,
Tamilnadu, India.
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Thirunelveli-627012
Tamil Nadu, India

ABSTRACT

Let G be a graph with minimum degree at least one. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number of G is denoted by χ_{td}(G) and is defined by the minimum number of colors needed in a total dominator coloring of G. In this paper, we obtain total dominator chromatic number on various classes of graphs.

MATHEMATICS SUBJECT CLASSIFICATION: **05C15, 05C69**

KEYWORDS : Total dominator chromatic number, banana graph ,book graph, stacked book graph, dutch wind mill graph, lollipop graph, gear graph, sunflower graph.

***Corresponding author**

Dr.A.Vijayalekshmi

Associate professor, Dept. of Mathematics,

S.T.Hindu College, Nagercoil-629002, India

Tamilnadu, India.

Email: vijimath.a@gmail.com

INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in[1]. Let G = (V, E) be a graph of order n with minimum degree atleast one . The open neighborhood N(v) of a vertex v∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v]= N (v)∪ {v}. An induced subgraph G[S], where S⊆V of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S. A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets $V_1$and V_2 such that no edge has both end points in the same subset, and each vertex of the first set is connected to every vertex of the second set and vice -verse. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and n-1 leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H , to be the graph with vertex set V(G)×V(H) and edges between two vertices (u_1 ,v_1) and (u_2,v_2) iff either $u_1=u_2$ and $v_1v_2 \in E(H)$ or $u_1u_2 \in E(G)$ and $v_1=v_2$.

A subset S of V is called a total dominating set if every vertex in V is adjacent to some vertex in S. The total dominating set is minimal total dominating set if no proper subset of S is a total dominating set of G. The total domination number γ_t is the minimum cardinality taken over all minimal total dominating set of G. A γ_t-set is any minimal total dominating set with cardinality γ_t.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G). A total dominator coloring (td- coloring) of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G. This concept was introduced by A.Vijiyalekshmi in[2]. This notion is also referred as a smarandachely k - dominator coloring of G (k≥1) and was introduced by A.Vijiyalekshmi in[3] . For an integer k≥1, a smarandachely k-dominator coloring of G is a proper coloring of G such that every vertex in G properly dominates a k color class. The smallest number of colors for which there exist a smarandachely k-dominator coloring of G is called the smarandachely k-dominator chromatic number of G, and is denoted by $\chi^{s}_{td}(G)$.

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C^* be a minimal td-coloring of G. We say that a color class $c_i \in C^*$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes. A banana graph $B_{m,n}$ is a graph obtained by connecting one leaf of each m copies of an n-star graph with a single root vertex that is distinct from all the stars . The book graph B_m is defined as the graph Cartesian product $P_2 \times K_{1,m-1}$. The stacked book graph $SB_{m,n}$ is the generalization of the book graph to stacked pages . The dutch windmill graph D_m^n is the graph obtained by taking n copies of the cycle graph C_n with a vertex in common .The lollipop graph $L_{m,n}$ is a graph consisting of a complete graph on m vertices and a path graph on n vertices connected with a bridge . The gear graph G_n is a wheel graph with a one single vertex added between each pair of adjacent vertices of the outer cycle. A sunflower graph Sf_n, where $n \geq 4$ is a graph obtained from n- cycle C_n by including a triangle on each outer edge so that one vertex of each outer triangle has degree 2.

The total dominator chromatic number of paths, cycles and ladder graphs were found in[4] .

We have the following observations from[4] .

Theorem A[4] . Let G be p_n or C_n. Then

$$\chi_{td}(p_n) = \chi_{td}(C_n) = \begin{cases} 2\left\lfloor \frac{n}{4} \right\rfloor + 2 & if\ n \equiv 0 (mod\ 4) \\ 2\left\lfloor \frac{n}{4} \right\rfloor + 3 & if\ n \equiv 1 (mod\ 4) \\ 2\left\lfloor \frac{n+2}{4} \right\rfloor + 2 & otherwise \end{cases}$$

In this paper, we obtain the least value for total dominator chromatic number on various classes of graphs.

Theorem 1 For the banana graph $B_{m,n}$ $\chi_{td}(B_{m,n})$=2m+1

Proof: Let $B_{m,n}$ be the banana graph .The vertex set of the graph $V(B_{m,n})$ =

{u} U $\{v_{ij} / 1 \leq i \leq n\ and\ 1 \leq j \leq m\}$ .T hat is ,$B_{m,n}$ consist of one vertex has degree m and m vertices of degree 2 and m vertices of degree n-1 and m(n-2) vertices of degree 1. We assign 2m distinct colors to degree 2 and n-1 respectively and the color say 2m+1 to the vertices of degree 1 and □ Thus $\chi_{td}(B_{m,n})$ = 2m+1.

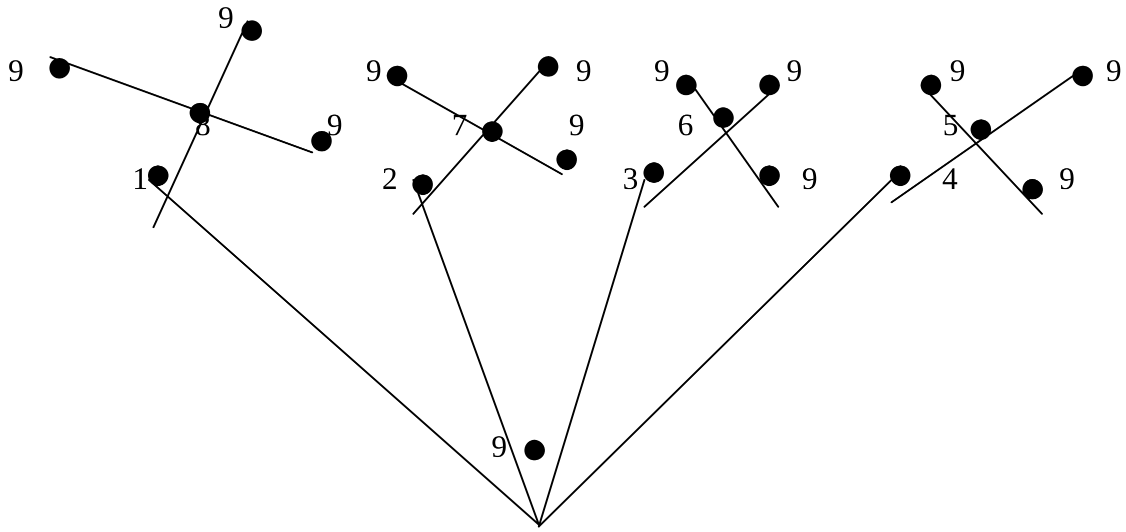
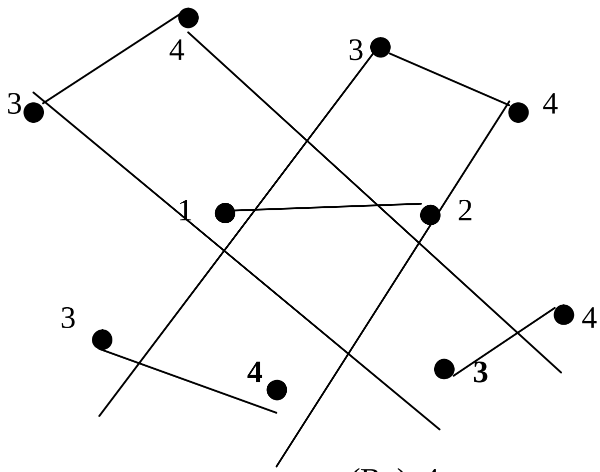


Fig 1 Banana Graph

$\chi_{td}(B_{4,5}) = 9$

Theorem 2 For the book graph B_m, $\chi_{td}(B_m)=4$

Proof : Let $P_2 \times K_{1,m}$ be the book graph with vertex set$\{v_1,v_2,v_3,\text{------},v_{2n},v_{2n+1},v_{2n+2}\}$, where (v_1,v_2) and (v_i, v_j) i=3,5,7------,2n+1 and j=4,6,8,------,2n+2 form the pages of B_m. We assign colors 1 and 2 to v_1 and v_2 repectively, assign the colors 3 and 4 to the set of vertices $\{v_3,v_5,v_7,\text{---------},v_{2n+1}\}$ and the set of vertices $\{v_4,v_6,v_8,\text{----},v_{2n+2}\}$ respectively. Thus $\chi_{td}(B_m) = 4$. □

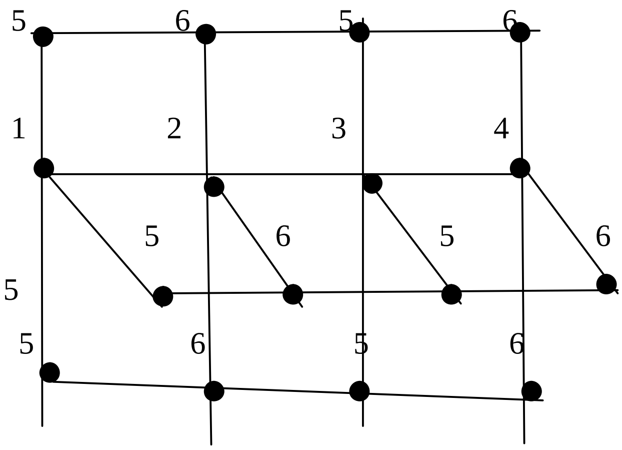


$\chi_{td}(B_4) = 4$

Fig 2 Book Graph

Theorem 3 For any stacked book graph $SB_{m,n}$, $\chi_{td}(SB_{m,n})=n+2$

Proof: Let $SB_{m,n}=P_n\times K_{1,m}$ be the stacked book graph and let $V(SB_{m,n})=$ $\{v_{ij} / 1 \le i \le n \text{ and } 1 \le j \le m\}$ such that B_i isomorphic to the vertex induced subgrarph v_{1i},v_{2i},v_{3i},---------,v_{ni}. We assign n distinct colors 1,2,3,----,n to v_{11},v_{21},v_{31},---------,v_{n1} and colors n+1 and n+2 to the set of vertices v_{ij}, $1\le j\le m$ and i=1,3,5-----,n if n is odd and the set of vertices v_{ij}, $1\le j\le m$ and i=2,4,6-----,n if n is even respectively.Thus $\chi_{td}(SB_{m,n})=n+2$.



$\chi_{td}(SB_{3,4})=6$

Fig 3 Stacked Book Graph

Theorem 4 For the dutch wind mill graph D_m^n ,

$$\chi_{td}(D_m^n)=\begin{cases} n\left(2\left\lfloor\frac{m-3}{4}\right\rfloor+3\right)-2n+4 & \text{if } m\equiv 0 \pmod 4\\ n\left(2\left\lfloor\frac{m-3}{4}\right\rfloor+2\right)-2n+4 & \text{if } m\equiv 3 \pmod 4\\ n\left(2\left\lfloor\frac{m-1}{4}\right\rfloor+2\right)-2n+4 & \text{otherwise}\end{cases}$$

Proof: Consider D_m^n formed by n copies of the cycle c_m with $V(D_m^n)=\{v_{ij}/_{j=1,2,3,------m}^{i=1,2,3,------n}\}$. For each i=1,2,3,---,n $\{v_{i1},v_{i2},v_{i3}$,---------,$v_{im}\}$ be the vertices of i- th copy of cycle C_m and $v_{11}=v_{21}=v_{31}$=--------- = v_{n1} is a common vertex. We assign color 1 and 2 to a common vertex v_{11} and the set of vertices $\{v_{i2}, v_{im}\}$, i=1,2,3----,n and we assign n $\chi_{td}(C_{m-3})$ distinct colors to remaining vertices $\{v_{i3},v_{i4},v_{i5}$,---------,$v_{i\,m-1}\}$,i=1,2,3,------,n. Totally we get n $\chi_{td}(C_{m-3})$ +2 colors to need td-coloring. We using repeated colour, so $\chi_{td}(D_m^n)$=n $\chi_{td}(C_{m-3})$ +2-2(n-1) .

Thus $\chi_{td}(D_m^n)= \begin{cases} n\left(2\left\lfloor\frac{m-3}{4}\right\rfloor+3\right)-2n+4 & if\ m\equiv 0\ (mod\ 4) \\ n\left(2\left\lfloor\frac{m-3}{4}\right\rfloor+2\right)-2n+4 & if\ m\equiv 3\ (mod\ 4) \\ n\left(2\left\lfloor\frac{m-1}{4}\right\rfloor+2\right)-2n+4 & otherwise \end{cases}$ □

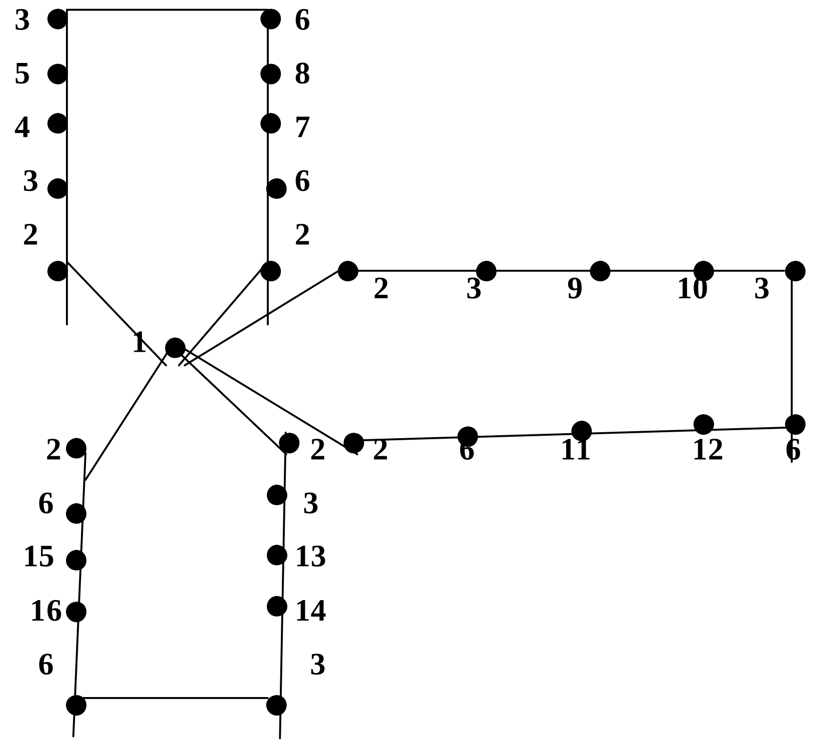


Fig 4 Dutch Wind mill Graph

$\chi_{td}(D_{11}^3)=16$

Theorem 5 For lollipop graph $L_{m,n}$,

$$\chi_{td}(L_{m,n}) = \begin{cases} 2\left\lfloor\frac{n-2}{4}\right\rfloor+2 & if\ n\equiv 2(mod\ 4) \\ 2\left\lfloor\frac{n-2}{4}\right\rfloor+3 & if\ n\equiv 3(mod\ 4) \\ 2\left\lfloor\frac{n}{4}\right\rfloor+2 & otherwise \end{cases}$$

Proof: Let $L_{m,n}$ be the lolli pop graph and let $V(L_{m,n})=\{ v_1,v_2,v_3,---------v_m ,v_{m+1},v_{m+2},v_{m+3},---------v_{m+n} \}$ be the set of vertex set, where the set of vertices $\{v_1,v_2,v_3,---------v_m\}$ form K_m and the set of vertices $\{v_{m+1},v_{m+2},v_{m+3},---------v_{m+n} \}$ form P_n and (v_m,v_{m+1}) is a bridge of $L_{m,n}$ .We assign colors 1 to the vertex set$\{v_m,v_{m+2}\}$ and the set of vertices $\{ v_1,v_2,v_3,---------v_{m-1} , v_{m+1}\}$ receive m distinct colors say 2,3,----m+1 respectively. Remaining (n-2) vertices $\{ v_{m+3},v_{m+4},v_{m+5},----------v_{m+n} \}$have $\chi_{td}(P_{n-2})$ colors for td –coloring.

$$\text{Thus } \chi_{td}(L_{m,n}) = \begin{cases} 2\left\lfloor \frac{n-2}{4} \right\rfloor + 2 & if\ n \equiv 2 (mod\ 4) \\ 2\left\lfloor \frac{n-2}{4} \right\rfloor + 3 & if\ n \equiv 3 (mod\ 4) \\ 2\left\lfloor \frac{n}{4} \right\rfloor + 2 & otherwise \end{cases}$$

□

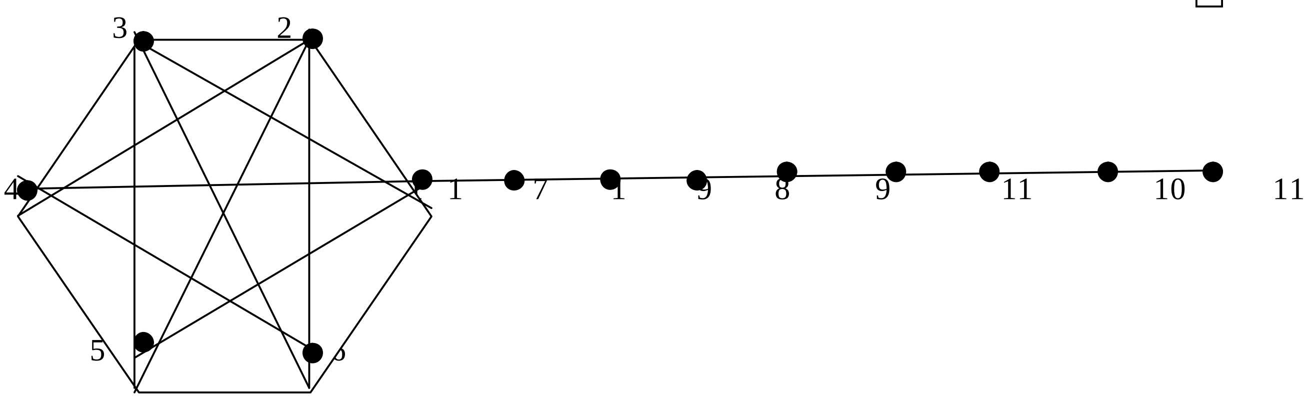
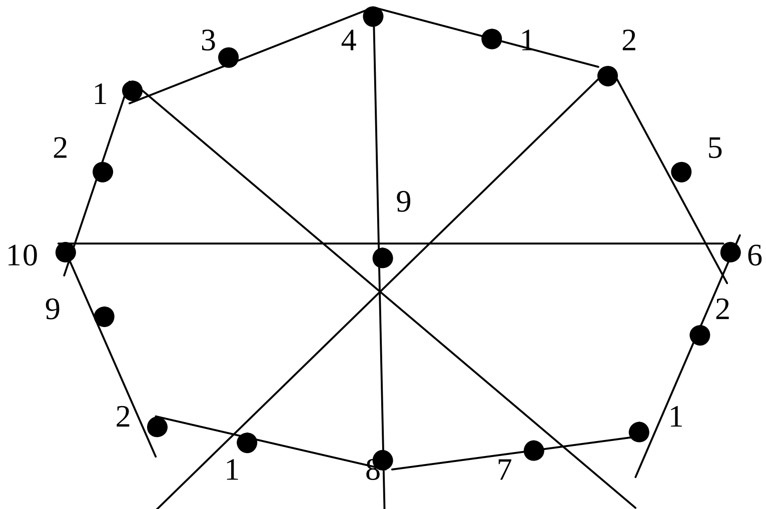


Fig 5 Lallipop Graph

$\chi td(L_{6,8})=11$

Theorem 6 Any gear graph G_n , $\chi_{td}(G_n)= \chi_{td}(C_{2n})$

Proof:: Let G_n be the gear graph with vertex set{ v_1,v_2,v_3,---------v_n ,v_{n+1},v_{n+2}, v_{n+3} ----------- v_{2n+1}},where v_1 is the central vertex and v_i $(2\leq i\leq 2n+1)$ be the vertices on the cycle C_{2n} . For td –coloring ,we need $\chi_{td}(C_{2n})$ colors for vertex set $\{v_i/ 2\leq i \leq 2n+1\}$and the central vertex receive any one of the above color. Thus $\chi_{td}(G_n) = \chi_{td}(C_{2n})$. □



$\chi_{td}(G_8)=10$

Fig 6.Gear Graph

Theorem 7 Any sunflower graph Sf_n , $\chi_{td}(Sf_n) = 1+\chi_{td}(C_n)$

Proof: Let Sf_n be the sun flower graph obtained taking a wheel with central vertex v_0 and the cycle C_n ($v_1v_2v_3$---------v_n v_1) and new vertices w_1, w_2, w_3,----,w_n where w_i is joined by the vertices v_i, v_{i+1}. We assign the color 1 to the set of vertices $\{v_0, w_1, w_2, w_3, ----, w_n\}$. Remaining vertices lies on the circle C_n, for td –coloring, we need $\chi_{td}(C_n)$ colors. Thus $\chi_{td}(Sf_n) = 1+\chi_{td}(C_n)$. □

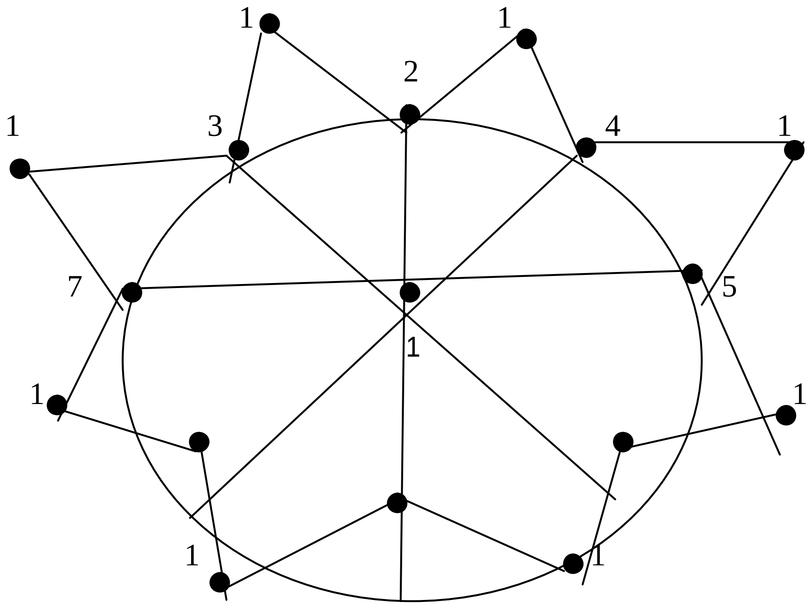


Fig 7 Sun Flower Graph

$\chi_{td}(Sf_8)$ =7

REFERENCES:

1. Harrary.F , Graph Theory ,Addition- wesley Reading, Mass, 1969.
2. Jinnah. M.I and Vijayalekshmi.A, Total dominator colorings in Graphs, Diss University of Kerala, 2010.
3. Vijayalekshmi.A, Total dominator colorings in Paths, International Journal of Mathematical Combinatorics, 2012;2:89-95.
4. Vijayalekshmi.A and Virgin Alangara sheeba.J ,Total dominator chromatic Number of paths, cycles and ladder graphs, 2018; 13(5):199 – 204.
5. Kavitha.K & David.N.G, dominator coloring of some classes of graphs, International jornal of Mathematical archive-2012; 3(11): 3954-3957.

6. Mojdeh,E.Nazari.D.A & Askari.S Total dominator chromatic Number in graphs, International conference on combinatorics, gritography and computation, 1997; 14C: 352-362.
7. Suganya.P ,Mary.R Jeya Jothi, Dominator chromatic number of some graph classes International Journal of Computational and Applied Mathematics,2017;12:458-463.
8. Kalaivani.R ,Vijayalakshmi.D, A Note on Dominator chromatic number of some graphs classes International Conference Applied and ComputationaMathematics, Conf.Series 1139,2017.
9. Sylvain Gravier, Total domination number of grid graphs, Discrete Applied Mathematics, 2002; 2: 119-128.
10. Terasa W.Haynes,Stephen T.Hedetniemi ,Peter J.Slater, Domination in Graphs, Marcel Dekker,NewYork,1998.
11. Terasa W.Haynes,Stephen T.Hedetniemi ,Peter J.Slater, Domination in Graphs – Advanced Topics, Marcel Dekker,NewYork,1998.