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Wheel Related Some Signed Product Cordial Graphs

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ABSTRACT

In this article, we discuss signed product cordial labeling of gear graph, flower graph, sunflower graph, cobweb graph and lotus inside a circle. We show that m times duplication of outer vertices of the flower graph as well as sunflower graph admits signed product cordial labeling. The graph obtained by duplication of all the vertices of degree two and the graph obtained by duplication of all the vertices of degree three in gear is signed product cordial graph.

KEYWORDS: Signed product cordial graph, sunflower graph, lotus inside a circle, flower graph, cobweb graph.

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INTRODUCTION

We begin with simple, finite, undirected graph $G = (V(G), E(G))$, where $V(G)$ and $E(G)$ are the vertex set and the edge set respectively. For all other terminology we follow Gross[1]. For latest survey of graph labeling we refer to Gallian[2], vast amount of literature is available of 200 different graph labeling techniques in 2600 or more papers. Now we provide brief summary of definitions and other information which are necessary for the present investigations.

Definition 1.1: A signed product cordial labeling of a graph G is a function $f: V(G) \rightarrow \{-1, 1\}$such that each edge uv is assigned the label $f(u)\, f(v)$, the number of vertices with label -1 andthe number of vertices with label 1 differ by atmost 1 and the number of edges with label -1 and thenumber of edges with label 1 differ by atmost 1. A graph which admits signed product cordial labeling is called a signed product cordial graph.

The notion of signed product cordial was introduced by Babujee and Loganathan[3], they investigated for path graph, cycle graph, star graph and other few graphs. Santhi and Kalidass[4] also proved that path union of flower, binary tree and star graph are signed product cordial graph. They further proved that path graph and $K_{1,n} \odot G_n$ are signed and total signed product cordial graphs.Few more results were given by Prajapati and Raval[5], they showed that few fractal graphs are signed product cordial graph.

Definition 1.2: A gear graph G_n is obtained from wheel graph$W_n = C_n + K_1$ by adding an vertex between every pair of adjacent vertices of C_n.

Definition 1.3: A helm graph H_n is obtained from wheel graph by adjoining a pendant edge on each rim vertex of the wheel.

Definition 1.4: A flower graph Fl_n is the graph obtained from a helm graph by joining each pendant vertex to the apex vertex of the helm.

Definition 1.5: The sunflower graph SF_n is the graph obtained by taking a wheel with apex vertex v_0 and the consecutive rim vertices $v_1, v_2, v_3, \ldots, v_n$and the additional vertices $u_1, u_2, u_3, \ldots, u_n$ where each u_i is joined by edges to v_i and $v_{i+1}, \forall\, i = 1, 2, 3, \ldots, n$ (where $v_{n+1}$is considered as v_1).

Definition 1.6: The graph lotus inside a circle LC_n is obtained from the cycle $C_n: v_1, v_2, \ldots, v_n, v_1$and a star $K_{1,n}$with center vertex v_0 and the end vertices $u_1, u_2, u_3, \ldots, u_n$by joining each $u_i$to v_i and $v_{i+1}, \forall\, i = 1, 2, 3, \ldots, n$ (where $v_{n+1}$is considered as v_1).

Definition 1.7: The spider's web or cobweb $Wb(m, n)$is a graph with m cycles $C_1, C_2, C_3, \ldots, C_m$where each $C_i$has consecutive vertices $v_{i,1}, v_{i,2}, \ldots, v_{i,n}, \forall\, i = 1, 2, \ldots, m$and corresponding vertices of the cyclesare joined by a path $v_{1,j}, v_{2,j}, \ldots, v_{m,j}, \forall\, j = 1, 2, \ldots, n$.

Definition 1.8: m times duplication of a vertex v of a graph G is the graph G' obtained from G by adding m new vertices $v_i, i = 1, 2, \dots, m$ to G such that $N_G(v) = N_{G'}(v_i), \forall\ i = 1, 2, \dots, m$.

MAIN RESULTS

Theorem 2.1: Gear graph is signed product cordial graph.

Proof: Let W_n be the wheel graph with the apex vertex v_0 and the consecutive rim vertices $v_1, v_2, \dots, v_n$. We obtain gear graph G_n by subdividing each of the rim edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1$ of the wheel graph by the vertices $u_1, u_2, \dots, u_{n-1}, u_n$ respectively. Obviously $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.

- Case 1: n is even.
  - Subcase 1: $n \equiv 2\ (mod\ 4)$

    Define a function $f: V(G) \to \{-1,\ 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = v_i, 1 \le i \le \frac{n}{2}; \\ -1, & if\ x = v_i, \frac{n}{2} \le i \le n; \\ 1, & if\ x \in \left\{u_{\frac{n}{2}}, u_n\right\}; \\ 1, & if\ x = u_i, i = 1, 3, 5, \dots, 2n-1, i \ne \left\{\frac{n}{2}, n\right\}; \\ -1, & if\ x = u_i, i = 2, 4, 6, \dots, 2n-2, i \ne \left\{\frac{n}{2}, n\right\}. \end{cases}$$

  - Subcase 2: $n \equiv 0\ (mod\ 4)$

    Define a function $f: V(G) \to \{-1,\ 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = v_i, 1 \le i \le \frac{n}{2}; \\ -1, & if\ x = v_i, \frac{n}{2} \le i \le n; \\ 1, & if\ x = u_n; \\ -1, & if\ x = u_{\frac{n}{2}}; \\ 1, & if\ x = u_i, i = 1, 3, 5, \dots, 2n-1, i \ne \left\{\frac{n}{2}, n\right\}; \\ -1, & if\ x = u_i, i = 2, 4, 6, \dots, 2n-2, i \ne \left\{\frac{n}{2}, n\right\}. \end{cases}$$

Thus for both the subcases we get $e_f(-1) = e_f(1) = \frac{n}{2}$. Hence $|v_f(-1) - v_f(1)| \le 1$ and $|e_f(-1) - e_f(1)| \le 1$.

- Case 2: n is odd.
  - Subcase 1: $n \equiv 1\ (mod\ 4)$

Define a function $f: V(G) \to \{-1,\ 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = v_i, 1 \leq i \leq \frac{n+1}{2}; \\ -1, & if\ x = v_i, \frac{n+3}{2} \leq i \leq n; \\ -1, & if\ x \in \left\{u_{\frac{n+1}{2}}, u_n\right\}; \\ 1, & if\ x = u_{\frac{n-1}{2}}; \\ 1, & if\ x = u_i, i \in \left\{1, 3, 5, \dots, \frac{n-3}{2}\right\} \cup \left\{n-1, n-3, \dots, \frac{n+3}{2}\right\}; \\ -1, & if\ x = u_i, i \in \left\{2, 4, 6, \dots, \frac{n-5}{2}\right\} \cup \left\{n-1, n-3, \dots, \frac{n+3}{2}\right\}. \end{cases}$$

- Subcase 2: $n \equiv 3\ (mod\ 4)$

Define a function $f: V(G) \to \{-1,\ 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = v_i, 1 \leq i \leq \frac{n+1}{2}; \\ -1, & if\ x = v_i, \frac{n+3}{2} \leq i \leq n; \\ -1, & if\ x \in \left\{u_{\frac{n+1}{2}}, u_n, u_{\frac{n+3}{2}}\right\}; \\ 1, & if\ x = u_i, i \in \left\{1, 3, 5, \dots, \frac{n-1}{2}\right\} \cup \left\{n-1, n-3, \dots, \frac{n+5}{2}\right\}; \\ -1, & if\ x = u_i, i \in \left\{2, 4, 6, \dots, \frac{n-3}{2}\right\} \cup \left\{n-1, n-3, \dots, \frac{n+7}{2}\right\}. \end{cases}$$

Thus, for both the subcases we get $e_f(1) = \left\lceil\frac{3n}{2}\right\rceil$ and $e_f(-1) = \left\lfloor\frac{3n}{2}\right\rfloor$Hence$\left|v_f(-1) - v_f(1)\right| \leq 1$ and $\left|e_f(-1) - e_f(1)\right| \leq 1$. Hence, by both the cases G_n admits signed product cordial labeling. So, $G_n$is signed product cordial graph.

Theorem 2.2: The graph obtained by duplication of all the vertices of degree three with an edge in gear graph $G_n, n > 2$ is signed product cordial graph.

Proof: Let W_n be the wheel graph with the apex vertex v_0 and consecutive rim vertices $v_1, v_2, v_3, \dots, v_n$.To obtain the gear graph G_n subdivide each of the rim edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1$ of the wheel graph by the vertices $u_1, u_2, \dots, u_{n-1}, u_n$ respectively. Obviously, $|V(G_n)| = 2n + 1$and $|E(G_n)| = 3n$. Let G be the graph obtained from $G_n$by duplicating each vertex v_i of degree three by an edge $v_i^l v_i^r$ respectively for all $i = 1, 2, 3, \dots, n$. Thus $|V(G)| = 4n + 1$ and $|E(G)| = 6n$.

Define a function $f: V(G) \to \{-1,\ 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = v_i, \forall\ i = 1,2,3,\dots,n; \\ 1, & if\ x = u_i, \forall\ i = 1,2,3,\dots,n; \\ -1, & if\ x \in \{v_i^l, v_i^r\}, \forall\ i = 1,2,3,\dots,n. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = \frac{n}{2}$. Hence $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$.Thus, the obtained graph G admits signed product cordial labeling.

Theorem 2.3: The graph obtained by duplication of all the vertices of degree two with an edge in gear graph $G_n, n > 2$ is signed product cordial graph.

Proof: Using the proof of theorem 2.2 we can prove that the graph obtained by duplication of all the vertices of degree two with an edge in gear graph $G_n$for $n > 2$ is signed product cordial graph.

Theorem 2.4: The sunflower is signed product cordial graph for $n > 2$.

Proof: The sunflower graph SF_n is the graph obtained by taking a wheel with apex vertex v_0 and the consecutive rim vertices $v_1, v_2, v_3, \dots, v_n$and the additional vertices $u_1, u_2, u_3, \dots, u_n$ where each u_i is joined by edges to v_i and v_{i+1} (where v_{n+1} is considered as v_1) respectively. Clearly, $|V(SF_n)| = 2n + 1$ and $|E(SF_n)| = 4n$.

Define a function $f: V(G) \rightarrow \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & if\ x = v_i, \forall\ i = 1,2,3,\dots,n; \\ -1, & if\ x = u_i, \forall\ i = 1,2,3,\dots,n. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = 2n$. Hence $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$.Hence, SF_n admits signed product cordial labeling.

Theorem 2.5: The graph obtained from sunflower graph by m times duplication of all the outer vertices$u_i, i = 1,2,\dots,n$ of SF_n by m vertices is signed product cordial graph when$m$ is even and $n > 2$.

Proof: Let G be the graph obtained from sunflower graph SF_n by duplicating each of the vertex u_i werename it as $\{u_i^0, i = 1,2,\dots,n\}$, m times by $\{u_i^j, 1 \leq i \leq n, 1 \leq j \leq m\}$ vertices. Clearly, $|V(G)| = 2mn + 1$and $|E(G)| = 2n(m + 2)$.Define a function $f: V(G) \rightarrow \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & if\ x = v_i, \forall\ i = 1,2,3,\dots,n; \\ -1, & if\ x = u_i^j, 1 \leq i \leq n, j = 0,2,4,\dots,m; \\ 1, & if\ u_i^j, 1 \leq i \leq n, j = 1,3,5,\dots,m-1. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = n(m + 2)$. Hence $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$.Thus, G admits signed product cordial labeling.

Theorem 2.6: Flower graph Fl_n is signed product cordial graph for $n > 2$.

Proof: A flower graph Fl_n is the graph obtained from a helm whose apex vertex is v_0 and the consecutive rim vertices $v_1, v_2, v_3, \dots, v_n$. We label all the pendant vertices as $u_i, 1 \le i \le n$ corresponding to each $v_i, 1 \le i \le n$. Fl_n is obtained by adding the edges $v_0 u_i, 1 \le i \le n$. Clearly, $|V(Fl_n)| = 2n + 1$ and $|E(Fl_n)| = 4n$. Define a function $f: V(G) \to \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & if\ x = v_i, \forall\ i = 1, 2, 3, \dots, n; \\ -1, & if\ x = u_i, \forall\ i = 1, 2, 3, \dots, n. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = 2n$. Hence $|v_f(-1) - v_f(1)| \le 1$ and $|e_f(-1) - e_f(1)| \le 1$.Hence, Fl_n admits signed product cordial labeling.

Theorem 2.7: The graph obtained from flower graph by m times duplication of all the outer vertices $u_i, i = 1, 2, \dots, n$ by m vertices is signed product cordial graph when, m is even and $n > 2$.

Proof: Using the proof of theorem 2.5 we can prove that the graph obtained from flower graph by m times duplication of all the outer vertices by a vertexis signed product cordial graph for even m and $n > 2$.

Theorem 2.8: The graph lotus inside a circle LC_n for $n > 2$ is signed product cordial graph.

Proof: The graph lotus inside a circle LC_n is obtained from the cycle $C_n: v_1, v_2, v_3, \dots, v_n, v_1$ and a star $K_{1,n}$ inside the cycle C_n withcenter vertex v_0 and the end vertices as $u_1, u_2, \dots, u_n$ by joining each u_i to v_i and v_{i+1} (where v_{n+1} is considered as v_1) respectively. Clearly, $|V(LC_n)| = 2n + 1$ and $|E(LC_n)| = 4n$.Define a function $f: V(G) \to \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_i, \forall\ i = 1, 2, 3, \dots, n; \\ 1, & if\ otherwise. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = 2n$. Hence $|v_f(-1) - v_f(1)| \le 1$ and $|e_f(-1) - e_f(1)| \le 1$.Hence, LC_n admits signed product cordial labeling.

Theorem 2.9: Cobweb $Wb(m, n), m > 2$ is signed product cordial graph for $n = 3, 4$ and 5.

Proof: Let cobweb $Wb(m, n) = G$. Clearly, $|V(G)| = nm$ and $|E(G)| = n(2m - 1)$. Define a function $f: V(G) \to \{-1, 1\}$ as follows:

Case 1: For $n = 3$,

$$f(x) = \begin{cases} -1, & if\ x = v_{2i-1,3}, i = 1, 2, 3, \dots, \left\lceil \frac{m}{2} \right\rceil; \\ 1, & if\ x = v_{2i-1,j}, i = 1, 2, 3, \dots, \left\lceil \frac{m}{2} \right\rceil, j = 1, 2; \\ -1, & if\ x = v_{2i,j}, i = 1, 2, 3, \dots, \left\lfloor \frac{m}{2} \right\rfloor, j = 1, 3; \\ 1, & if\ x = v_{2i,2}, i = 1, 2, 3, \dots, \left\lfloor \frac{m}{2} \right\rfloor. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = \left\lceil \frac{3(2m-1)}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{3(2m-1)}{2} \right\rfloor$.

Case 2: For $n = 4$,

$$f(x) = \begin{cases} 1, & if\ x = v_{2i-1,j}, i = 1,2,3,\dots,\left\lceil\frac{m}{2}\right\rceil, j = 1,2; \\ -1, & if\ x = v_{2i-1,j}, i = 1,2,3,\dots,\left\lceil\frac{m}{2}\right\rceil, j = 3,4; \\ 1, & if\ x = v_{2i,j}, i = 1,2,3,\dots,\left\lfloor\frac{m}{2}\right\rfloor, j = 1,4; \\ -1, & if\ x = v_{2i,j}, i = 1,2,3,\dots,\left\lfloor\frac{m}{2}\right\rfloor, j = 2,3. \end{cases}$$

Thus, by using above definition we get $e_f(-1) = e_f(1) = 4m - 2$.

Case 3: For $n = 5$,

$$f(x) = \begin{cases} 1, & if\ x = v_{2i-1,j}, i = 1,2,3,\dots,\left\lceil\frac{m}{2}\right\rceil, j = 1,2,3; \\ -1, & if\ x = v_{2i-1,j}, i = 1,2,3,\dots,\left\lceil\frac{m}{2}\right\rceil, j = 4,5; \\ 1, & if\ x = v_{2i,j}, i = 1,2,3,\dots,\left\lfloor\frac{m}{2}\right\rfloor, j = 3,4; \\ -1, & if\ x = v_{2i,j}, i = 1,2,3,\dots,\left\lfloor\frac{m}{2}\right\rfloor, j = 1,2,5. \end{cases}$$

Thus, by using above definition we get $e_f(1) = \left\lceil\frac{5(2m-1)}{2}\right\rceil$ and $e_f(-1) = \left\lfloor\frac{5(2m-1)}{2}\right\rfloor$.Hence, by all the three cases cobweb graph is signed product cordial graph for $n = 3,4,5$.

CONCLUSION

We conclude that few graphs like gear graph, the graph obtained by duplication of all the vertices of degree two and the graph obtained by duplication of all the vertices of degree three in gear are signed product cordial graphs. Also, flower graph, sunflower graph, lotus inside a circle and cobweb graph for $n = 3,4,5$ admits signed product cordial labeling. The graph obtained by m times duplication of outer vertices of flower graph and sunflower graph, for even m are also signed product cordial graph.

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