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H-Recurrent Finsler Connection

Brijendra Krishna Singh

Department of Mathematics Amity University Chhattisgarh, Raipur, India
Email. : bks0509@gmail.com

ABSTRACT

The Decomposition of the normal Finsler connection tensor N^i_{jkh} of a finsler connection in the form of H Recurrent Finsler Connection and assume that decompose vector field X^i is not independent of directional arguments then thenormal projective curvature tensor are connected by recurrent Finsler connection.

KEYWORDS: *Finsler, manifolds, torsion, projective, recurrence*

***Corresponding author:**

Brijendra Krishna Singh

Department of Mathematics

Amity University Chhattisgarh,

Raipur, India

Email. : bks0509@gmail.com

INTRODUCTION:

A Finsler manifold F_n of dimension n is a manifold F_n associated with a fundamental function $F(x,\dot{x})$, the metric tensor of (F_n, F) is given by

(1.1) $g_{ij} = \frac{1}{2}\dot{\partial}_i \dot{\partial}_j F^2$ where $\dot{\partial}_i = \partial / \partial_{\dot{x}^i}$.

A Finsler connection of (F_n, F) is a triad $(F^i_{jk}, N^i_k, C^i_{jk})$ of a v-connection F^i_{jk}, a nonlinear connection N^i_k and a vertical connection C^i_{jk} [6]. The h- and v- covariant derivatives of any tensor field V^i_j corresponding to a given Finsler connection is given by

(1.2) $V^i_{j|k} = d_k V^i_j + V^m_j F^i_{mk} - V^i_m F^m_{jk}$,

(1.3) $V^i_{j|k} = \partial_k V^i_j + V^m_j C^i_{mk} - V^i_m C^m_{jk}$

where (1.4) $d_k = \partial_k - N^m_k \partial_m,\ \partial_k = \partial / \partial_{xk}$.

From a given Finsler metric we can determine various Finsler connections. In the present studies we shall use the Cartan connection which will be denoted by $C\Gamma : (\Gamma^{-xi}_{jk}, G^i_k, C^i_{jk})$. These connections can be uniquely determined from the metric function F by the following axioms:

(A_1) The connection is h – metrical i.e. $g_{ij}/k = 0$,

(A_2) The connection is v – metrical i.e. $g_{ij}/k = 0$,

(A_3) The deflection tensor field D^i_k vanishes,

(A_4) The (h) h – torsion tensor field T^i_{jk} vanishes,

(A_5) The (v) v – torsion tensor field S^i_{jk} vanishes.

All these five axioms have been mentioned in [7]. The individual members of the triad are given as

(1.13) $\Gamma^{xi}_{jk} = \frac{1}{2} g^{ih} (d_k g_{jh} + d_j g_{kh} - d_h g_{jk})$,

(1.14) $a)\ G^i_k = \partial_k G^i = \gamma^i_{ok} - 2C^i_{km} G^n$,

$b)\ G^i = \frac{1}{2}\gamma^i_{oo}$,

(1.15) $C^i_{j|k} = g^{ih} C_{jhk}, \quad C_{jhk} = \frac{1}{2}\partial_h g_{jk}$,

where (1.16) $\gamma^i_{jk} = \frac{1}{2} g^{ih} (\partial_k g_{jh} + \partial_j g_{kh} - \partial_h g_{jk})$,

DEFINITION (1.1):

A Finsler connection will be called h-recurrent Finsler connection $RF\Gamma$ if it satisfies the following axioms:

$(A_1)'$ The connection is h-recurrent with recurrence vector α_k i.e. $g_{ij|k} = \alpha_k\, g_{ij}$.

$(A_2)'$ The connection is v-metrical i.e. $g_{ij|k} = 0$.

$(A_3)'$ The deflection tensor field is given by D^i_k.

$(A_4)'$ The (h) h-torsion tensor field T^i_{jk} vanishes.

$(A_5)'$ The (v) v-torsion tensor field S^i_{jk} vanishes.

In view of equations (1.18), (1.20) and (1.22) we find that the h-recurrent Finsler connection $RF\Gamma$ are given by

(1.23) $F^i_{jk} = \overset{c}{F}{}^i_{jk} - C^i_{km}\, X^m_j - C^i_{jm}\, X^m_k + C_{jkm} X^{mi},$

(1.24) $N^i_k = \overset{c}{N}{}^i_k + X^i_k,$

(1.25) $C^i_{jk} = \overset{c}{C}{}^i_{jk} = \dfrac{1}{4} g^{ih}\, \dot{\partial}_h\, \dot{\partial}_j\, \dot{\partial}_k\, F^2$

Where (1.26) $X^i_k = C^i_{km}\, B^m_o - B^i_k,$

(1.27) $B^i_k = D^i_k + \dfrac{1}{2}(\alpha_o\, \delta^o_k + \alpha_k\, \dot{x}^i - \alpha^i y_k)$

(1.28) $X^{mi} = g^{ji}\, X^m_j$

and $\left(\overset{c}{F}{}^i_{jk}, \overset{c}{N}{}^i_k, \overset{c}{C}{}^i_{jk} \right)$ are the coefficients of Cartan connection $C\Gamma$. With the help of the equations (1.8), (1.23) and (1.24) the (v) hv –torsion tensor $RF\Gamma$ can be written as

(1.29) $P^i_{jk} = \overset{c}{P}{}^i_{jk} + X^i_j\big| k + C^i_{jm}\, X^m_k + C_{jkm}\, (X^{im} - X^{mi})$

where P^i_{jk} is the (v) hv-torsion tensor of Cartan connection $C\Gamma$ and $|$ means v-covariant differentiation with respect to $C\Gamma$ or $RF\Gamma$. Again using the equations (1.7) and (1.24), we get the following alternative form of (v) hv-torsion tensor of $RF\Gamma$.

(1.30) $R^v_{jk} = \overset{c}{R}{}^i_{jk} - \overset{c}{P}{}^i_{jm}\, X^m_k + \overset{c}{P}{}^i_{km}\, X^m_j + X^i_j + C_{|k}$

$$- X^i_k\, C_{|j} - X^m_k\, X^i_j\big|_m + X^m_j\, X^i_k\big|_m - C^i_{jm}\, X^i_r\, X^m_k + C^r_{km}\, X^i_r\, X^m_j$$

THE (v) hv-TORSION TENSOR OF THE FORM $P^i_{jk} = -\,\dot{\delta}_k B^i_j$

In this section we shall pay our attention to that h-recurrent Finsler connection $RF\Gamma$ whose (v) hv-torsion tensor P^i_{jk} is being expressed by the following equation

$$(4.1) P^i_{jk} = -\dot{\delta}_k B^i_j,$$

where B^i_j is the tensor field of the Finsler connection (1.27). Using (4.11) in (1.29), we get

$$(4.2)\ \overset{c}{P}{}^i_{jk} = \dot{\delta}_k (C^i_{jr} B^r_0) + C^i_{mk} X^m_j + C^i_{jm} X^m_k - C_{jkm} X^{mi} = 0.$$

Using $\dot{\delta}_k g_{ij} = 2C_{ijk}$ in (4.2), we get

$$(4.3)\ \overset{c}{P}_{ijk} + \dot{\delta}_k (C_{ijr} B^r_0) - 2C_{irk} C^r_{jm} B^m_0 + C_{imk} X^m_k +$$

$$+ C_{ijm} X^m_k - C_{jkm} X^m_i = 0.$$

Since C_{ijk} and $\overset{c}{P}_{ijk}$ are symmetric in *i* and *j*, hence from (4.3), we get

$$(4.4)\ S_{ijmk}\ B^m_0 C_{imk} X^m_j - C_{jmk} X^m_i = 0.$$

Multiplying (4.4) by $\dot{x}^i$, we get

$$(4.5)\ C_{jmk} X^m_0 = 0.$$

An obvious of (4.5) is the equation

$$(4.6)\ X^i_j = -B^i_j \text{ and } C_{ikm} B^m_j = C_{jkm} B^m_i.$$

In the light of these observations from (4.3), we get

$$(4.7)\ \overset{c}{P}_{ijk} = C_{jkm} B^m_i.$$

Substituting these results into the equations (1.30), (1.31) and (1.32), we get

$$(4.8)\ R^i_{jk} = \overset{c}{R}{}^i_{jk}\ B^i_j C_{|k} + B^i_k C_{|j} - B^m_k\ B^i_j\Big|_m + B^m_j\ B^i_k\Big|_m,$$

$$(4.9)\ P^i_{hjk} = \overset{c}{P}{}^i_{hjk}\ S^i_{hjk} B^r_j,$$

and $$(4.10)\ R^i_{hjk} = \overset{c}{R}{}^i_{hjk} + \overset{c}{P}{}^i_{hjm}\ B^m_k - \overset{c}{P}{}^i_{hkm}\ B^m_j + S^i_{hrs}\ B^r_j\ B^s_k.$$

If we now assume that

$$(4.11)\ P^c_{ijk} = C_{jkm} B^m_i \text{ holds,}$$

then this assumption gives

$$(4.12)\ C_{ijr} B^r_k = C_{ikr} B^r_j,\ C_{ijr} B^r_0 = 0 \text{ and } X^i_j = -B^i_j.$$

Using (4.12) in (1.27), we get

$$(4.13)\ P^i_{jk} = -\dot{\delta}_k B^i_j.$$

Therefore, we can state.

THEOREM (4.1):

If F_n be supposed to be an n-dimensional Finsler space equipped with h-recurrent Finsler connection $RF\Gamma$ and with the deflection tensor D^i_j and recurrence vector α_k, if we further suppose that $B^i_j = D^i_j + \frac{1}{2}(\alpha_0 \delta^i_j + \alpha_j \dot{x}^i - \alpha^i y_j)$ then the (v) hv-curvature tensor P^0_{jk} of $RF\Gamma$ is given by $P^i_{jk} = -\dot{\delta}_k D^i_j$ if and only if the (v) hv-torsion tensor $\overset{c}{P}{}^i_{jk}$ of the connection $C\Gamma$ is represented by $\overset{c}{P}{}^i_{jk} = C^i_{jm} B^m_k$ and in such a case the (v) h-torsion tensor R^i_{jk} of the hv-curvature tensor P^i_{hjk} and the h-curvature tensor R^i_{hjk} of recurrent Finsler connection $RF\Gamma$ are respectively given by (4.8), (4.9) and (4.10).

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