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MHD Chemically Reacting Viscoelastic Fluid Past an Impulsively Started Infinite Vertical Plate with Heat and Mass Transfer Effect

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ABSTRACT

In the present study, we discus the heat and mass transfer on the unsteady viscoelastic second order Rivlin-Erickson fluid past an impulsively started infinite vertical plate in the presence of a foreign mass and constant mass flux on taking into account of viscous dissipative heat at the plate under the influence of a uniform transverse magnetic field in the presence of chemical reaction. The flow is governed by a coupled non-linear system of partial differential equations. The velocity, the temperature, the concentration, the skin-friction, the rate of heat transfer and Sherwood number are obtained by using Crank Nicolson finite difference method. Numerical results are graphically discussed for various values of physical parameters of interest.

KEYWORDS: Mass Transfer effects on MHD Viscous Flow, Rivlin-Erickson Fluid, Mass flux, foreign mass, Non-linear Systems and chemical reaction

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INTRODUCTION

The study of viscoelastic fluids had become of increasing importance in the last few years. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic films etc. When the manufacturing process at high temperature and need cooling the stretching sheet. The flows may need visco-elastic fluids to produce a good effect to reduce the temperature from the sheet. Engineering processes in which a fluid supports an exothermal chemical or nuclear reaction are very common today and the correct process design requires accurate correlation for the heat transfer coefficients at the boundary surfaces. Despite its increasing importance in technological and physical problems, the unsteady MHD free convection flows of dissipative fluids past an infinite plate have received much attention because of non-linearity of the governing equations. Without taking into account viscous dissipative heat and MHD, this problem was solved by Siegal [18] by integral method. The experimental confirmations of these results were presented by Goldstein and Eckert [9]. Other papers in this field are by Gebhart[7], Schetz and Eichhorn [17], Monold and Yang [13], Chung and Anderson [2], Goldstein and Briggs 8, etc. In all these papers, the effect of viscous dissipative heat and MHD was assumed to be neglected. Flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate was first studied by Stokes [24]. Stewartson [23] presented analytic solution to the viscous flow past an impulsively started semi-infinite horizontal plate whereas Hall [10] solved the same problem by finite difference method. Soundalgekar[21] first presented an exact solution to the flow of a viscous incompressible fluid past an impulsively started infinite vertical plate by the Laplace transform technique. The effect of the presence of impurities is studied in scientific literature by considering it as a foreign mass. It is usually a very complicated phenomenon; however, by introducing suitable assumptions, the governing equations can be simplified. Ganesan and palani [4] have studied Finite Difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass transfer. More recently; Satyanarayana *et.al* [16] Viscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving porous plate.

A researcher with a wide interest, started studies in compressible flow mostly from a mathematical approach. At that time there wasn't the realization that the flow could be choked. It seems that Rayleigh was the first who realized that flow with chemical reactions (heat transfer) can be choked. Muthucumaraswamy and Ganesan [14] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka *et al.* [3] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Soundalgekar and Patti [20] studied the problem of the flow past an impulsively started isothermal infinite vertical plate with mass transfer effects. The effect of foreign mass on the free-convection flow past a semi-infinite vertical plate was studied by Gebhart [5]. Chamkha [1] assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Raptis [15] investigate the steady flow of a viscous fluid through a porous medium bounded by a porous plate subjected to a constant suction velocity by the presence of thermal radiation. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [25]. More recently; Kesavaiah *et.al* [12] effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction.

Eckert developed the understanding of heat dissipation in relation to kinetic energy, especially in compressible flow. In most of the studies mentioned above, viscous dissipation is neglected. Gebhart and Mollendorf [6] considered the effects of viscous dissipation for external natural convection flow over a surface. Sparrow [22] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cookey *et al.* [11] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Karunakar Reddy *et.al.* [26] MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction.

The objective of the present paper is to analyze the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux, on taking into account of viscous dissipative heat under the influence of a uniform transverse magnetic field with heat source in the presence of chemical reaction. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using Crank Nicolson finite difference method. The behaviour of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

FORMULATION AND SOLUTION OF THE PROBLEM

Consider the flow of a viscous incompressible viscoelastic second order Rivlin-Erickson fluid past an impulsively started infinite vertical plate with heat source in the presence of chemical reaction. The x' – axis is taken along the plate in the vertically upward direction and the y'- axis is chosen normal to the plate. Initially the temperature of the plate and the fluid $T_\infty{}'$, and the species concentration at the plate $C_w{}'$ and in the fluid throughout $C_\infty{}'$ are assumed to be the same. At time $t' > 0$, the plate temperature is changed to $T_w{}'$ causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate and the plate starts moving upward due to impulsive motion, gaining a velocity of U_0. A uniform magnetic field of intensity H_0 is applied in the y-direction. Therefore the velocity and the magnetic field are given by $\bar{q} = (u, 0, 0)$ and $H = (0, H_0, 0)$ The flow being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow and Cess [22]) in the absence of any input electric field, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} + K_0{}^*\frac{\partial^3 u'}{\partial y'^2 \partial t'} - \frac{\nu}{K'}u' - \frac{\sigma\mu_e^2 H_0{}^2}{\rho}u' \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 - Q_0(T' - T'_\infty) \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'(C' - C'_\infty) \quad (3)$$

The initial and boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned} &u' = 0_p, T' = T'_\infty 0, C' = C'_\infty && \textit{for all} \quad y'\ t' > 0 \\ &u' = U_0, T' \to T'_w, \frac{dC'}{dy} = -\frac{j''}{D} && at \quad y' = 0 \\ &u' = 0, T' \to T'_\infty, C' \to C'_\infty && as \quad y \to \infty \end{aligned} \quad (4)$$

Where u' is the velocity of the fluid along the plate in the x'- direction, t' is the time, g is the acceleration due to gravity, β is the coefficient of volume expansion, β^* is the coefficient of thermal expansion with concentration, T'_∞ is the temperature of the fluid near the plate, T'_w is the temperature of the fluid far away from the plate, T'_w is the temperature of the fluid, C' is the species concentration in the fluid near the plate, C'_∞ is the species concentration in the fluid far away from the plate, j'' is the mass flux per unit area at the plate, ν is the kinematic viscosity, K_0^* is the coefficient of kinematic visco-elastic parameter, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, H_0 is the strength of applied magnetic field, ρ is the density of the fluid, C_p is the specific heat at constant pressure, K is the thermal conductivity of the fluid, μ is the viscosity of the fluid, D is the molecular diffusivity, U_0 is the velocity of the plate.

Equations (1) - (3) can be made dimensionless by introducing the following dimensionless variables and parameters:

In order to write the governing equations and the boundary conditions in dimensionless from, the following non-dimensional quantities are introduced.

$$u=\frac{u'}{U_0}, y=\frac{U_0 y'}{\nu}, t=\frac{t'U_0{}^2}{\nu}, \theta=\frac{T'-T_\infty{}'}{T'_w-T_\infty{}'}, \lambda=\frac{K_0{}^*U_0{}^2}{\nu^2},\ Kr=\frac{Kr'\nu}{U_0{}^2}$$
$$C=\frac{C'-C'_\infty}{j''/DU_0},\quad K=\frac{K'U_0{}^2}{\nu^2},\ \Pr=\frac{\nu\rho C_p}{K},\ Sc=\frac{\nu}{D},\ \phi=\frac{\nu Q_0}{\rho C_p U_0{}^2} \tag{5}$$
$$M=\frac{\sigma\mu_e^2 H_0{}^2\nu}{\rho U_0{}^2},\qquad Gr=\frac{\nu\beta g\left(T'_w-T'_\infty\right)}{U_0{}^3},\qquad Gc=\frac{\nu\beta^* g\ j''/DU_0}{U_0{}^3}$$

where Gr is the thermal Grashof number, Gc is modified Grashof Number, $\Pr$ is Prandtl Number, M is the magnetic field, Sc is Schmdit number, Kr is Chemical Reaction, K is Porous Permeability, ϕ is Heat source parameter respectively.

In terms of the above dimensionless quantities, Equations (1) - (2) reduces to

$$\frac{\partial u}{\partial t}=Gr\,\theta+Gc\ C+\frac{\partial^2 u}{\partial y^2}+\lambda\left(\frac{\partial^3 u}{\partial y^2\partial t}\right)-\frac{1}{K}u-M\,u \tag{6}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2-\phi\theta \tag{7}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-Kr\,C \tag{8}$$

The corresponding boundary conditions are

$$
\begin{aligned}
& u=0, \theta=0, C=0 \qquad for\ all \quad y, t \le 0 \\
& u=1, \theta=1, \frac{dC}{dy}=-1 \qquad at\ y=0 \\
& U=0, \theta \to 0, C \to 0 \qquad as\ y \to \infty
\end{aligned} \tag{9}
$$

SOLUTION OF THE PROBLEM

Equation (6) – (8) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (9). However, exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (6) - (8) are as follows:

$$
\begin{aligned}
\left[\frac{u_{i,j+1}-u_{i,j}}{\Delta t}\right] &= Gr\left[\theta_{i,j}\right]+Gc\left[C_{i,j}\right]+\left[\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right] \\
&\quad +\lambda\left[\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}-u_{i-1,j}+2u_{i,j}-u_{i+1,j}}{\Delta t.(\Delta y)^2}\right]-\frac{1}{K}\left[u_{i,j}\right]-M\left[u_{i,j}\right]
\end{aligned} \tag{10}
$$

$$
\Pr\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right]=\left[\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right]+\Pr Ec\left[\frac{u_{i,j+1}-u_{i,j}}{\Delta y}\right]-\Pr\phi\left[\theta_{i,j}\right] \tag{11}
$$

$$
Sc\left[\frac{C_{i,j+1}-C_{i,j}}{\Delta t}\right]=\left[\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right]-Kr\,Sc\left[C_{i,j}\right] \tag{12}
$$

Here, index i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$.

From the initial condition in (9), we have the following equivalent:

$$
u(i,0)=0, \theta(i,0)=0, C(i,0)=0 \quad for\ all\ \ i \tag{13}
$$

The boundary conditions from (9) are expressed in finite-difference form as follows

$$
\begin{aligned}
& u(0,j)=1, \theta(0,j)=1, C_{i-1,j}-C_{i+1,j}=-2 \quad for\ all \quad j \\
& u(i_{\max},j)=0, \theta(i_{\max},j)=0, C(i_{\max},j)=0 \quad for\ all \quad j
\end{aligned} \tag{14}
$$

Here $i_{\max}$ was taken as 50

First the velocity at the end of time step viz $u(i, j+1)(i=1,50)$ is computed from (10) in terms of velocity, temperature and concentration at points on the earlier time-step. Then $\theta(i, j+1)$ is computed from (11) and $C(i, j+1)$ is computed from (12). The procedure is repeated until t □□0.5 (i.e. j □□500). During computation, it was chosen as 0.001.

To judge the accuracy of the convergence and stability of finite difference scheme, the same program was run with different values of □t i.e., □t □□0.0009, 0.0001 and no significant change was observed. Hence, we conclude that the finite-difference scheme is stable and convergent.

Skin-friction:

We now calculate Skin-friction from the velocity field. It is given in non-dimensional form as:

$$\tau = -\left(\frac{du}{dy}\right)_{y=0}, \text{ where } \tau = -\frac{\tau'}{\rho U_0^{\,2}}$$

Numerical values of τ are calculated by applying the Newton's forward interpolation formula for five points.

Rate of heat transfer:

The dimensionless rate of heat transfer is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

Sherwood number:

The dimensionless Sherwood number is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

RESULT AND DISCUSSION

Figure (1), shows that the velocity distribution (u) is drawn against y for different values of magnetic parameter (M) for visco-elastic parameter $\lambda = 0.1$ and Prandtl number $(\Pr) =$ 0.71 (air). We notice that the velocity distribution (u) decreases with the increase in (M). From figure (2), velocity distribution (u) for visco-elastic parameter $\lambda = 0.1$ is drawn against y for different values of Grashof number (Gr). We notice that the (u) increases as Gr increases. From figure (3), velocity distribution (u) is drawn against y for different values of visco-elastic parameter λ. Here velocity distribution (u) increases with the increase of λ. Figures (4) and (5) represents the distribution (u) is drawn against y for different values of Schmdit number (Sc) and Eckert number (Ec). We observe that velocity distribution (u) is decrease with increase in Sc, where as the reverse effect noticed with increase Ec. Figures (6) and (7) represents the distribution (u) is drawn against y for different values of porous permeability parameter (K) and Chemical reaction parameter (Kr). We observe that velocity distribution (u) is increase with increase in K, where as decrease with increase Kr

Figures (8) - (14) is drawn for Temperature distribution (θ) against y for different values of Eckert number (Ec), Groshof number (Gr), Porous permeability parameter (K), Chemical reaction parameter (Kr), Magnetic parameter (M), heat source parameter (ϕ) and Schmidt number (Sc). Here we observed that the Temperature distribution (θ) increases with Eckert number (Ec) and heat source parameter (ϕ) increases, where as the reverse effect noticed with increase in Gr, K, Kr, M and Sc.

From figures (15) and (16) we observe that an increase in Kr and Sc leads to decrease in the Concentration profiles (C).

Table 1: Values of the Schmidt number Sc

Pr	Spices	Sc
0.71	Hydrogen (H)	0.24
	Helium (He)	0.30
	Water (H_20)	0.60
	Oxygen	0.66
	Ammonia (NH_3)	0.78
	Carbon dioxide	0.94
	Carbon dioxide (CO_2)	1.00
	Ethyl benzene $(C_6H_5CH_2CH_3)$	2.00

Figure (17) and Figure (18) is drawn for Skin-friction (τ) and Nusselt number (Nu) is drawn against magnetic parameter (M) and Prandtl number $(\Pr)$ for different values of chemical reaction parameter (Kr) and Grashof number (Gr) . From these figures we observe that τ decreases with the increase of Kr for Pr=0.71, but Nusselt number (Nu) increases with the increase of Gr.

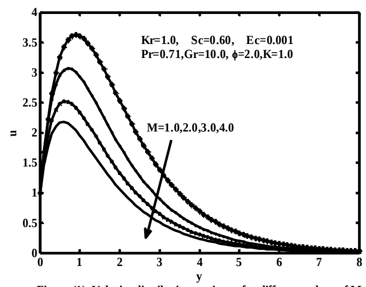

Figure (1): Velocity distribution against y for different values of M

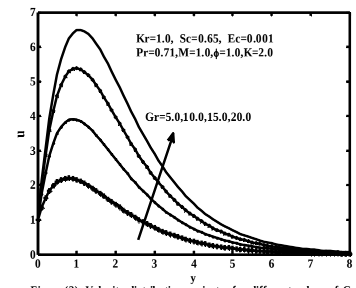

Figure (2): Velocity distribution against y for different values of Gr

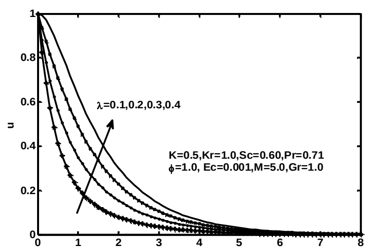

Figure (3). Velocity distribution against y for different values of λ

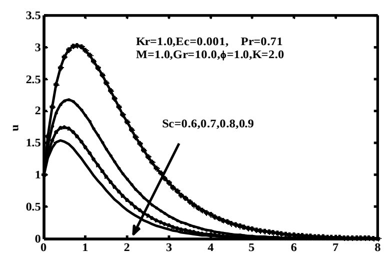

Figure (4): Velocity distribution against y for different values of Sc

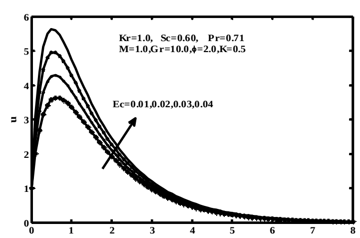

Figure (5): Velocity distribution against y for different values of Ec

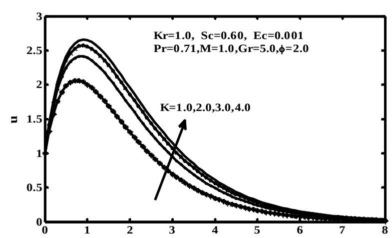

Figure (6): Velocity distribution against y for different values of K

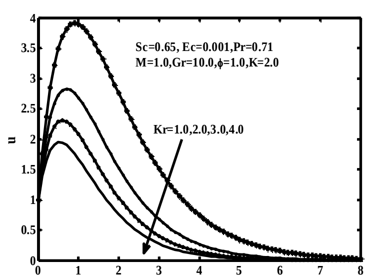

Figure (7): Velocity profiles against y for different values of Kr

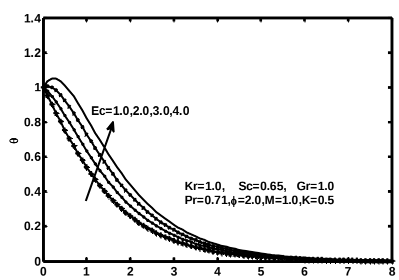

Figure (8): Temperature distribution against y for different values of Ec

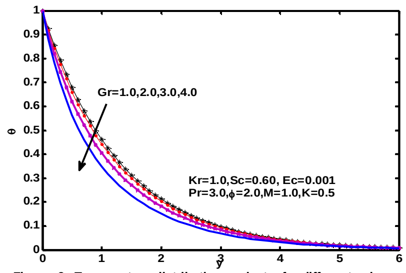

Figure. 9. Temperature distribution against y for different values of Gr

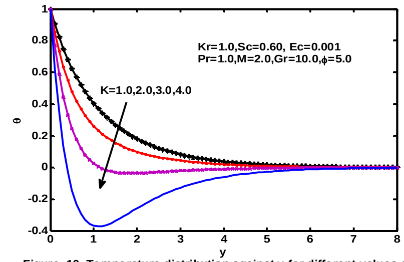

Figure. 10. Temperature distribution against y for different values of K

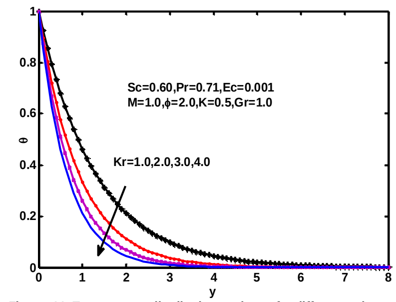

Figure. 11. Temperature distribution against y for different values of Kr

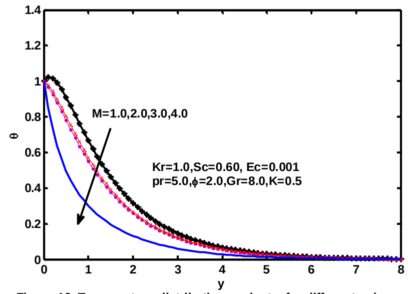

Figure 12. Temperature distribution against y for different values of M

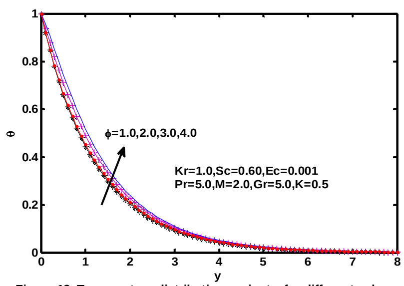

Figure 13. Temeprature distribution against y for different values of ϕ

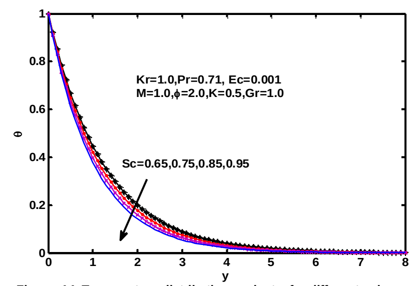

Figure. 14. Temperature distribution against y for different values of Sc

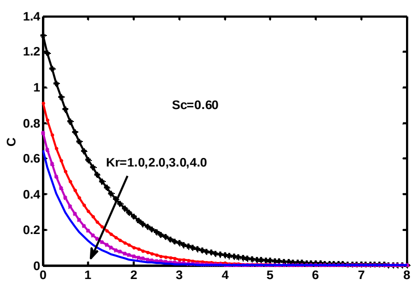

Figure. 15. Concentration distribution against y for different Kr

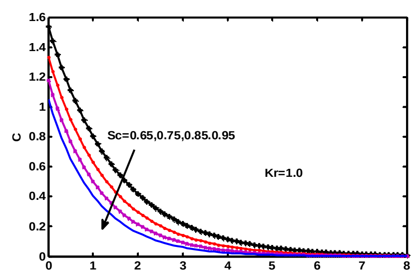

Figure. 16. Concentration distribution against y for different values of Sc

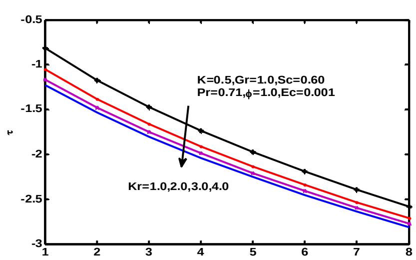

Figure.17.Skin-friction against M for different values of Kr

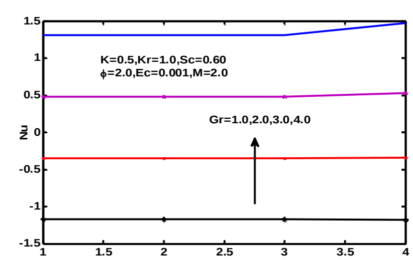

Figure 18. Nusselt number against Pr for different values of Gr

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