Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Fuzzy Biclosed Maps And Pairwise Fuzzy Biclosed Maps In Fuzzy Biclosure Spaces

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ABSTRACT

The objective of this paper is to introduceconcept of fuzzy biclosed(fuzzy biopen) maps and pairwise fuzzy biclosed maps(pairwise fuzzy biopen maps) in fuzzy biclosure spaces. Some important properties and characterization of fuzzy biclosed (fuzzy biopen) maps and pairwise fuzzy biclosed (pairwise fuzzy biopen) maps in fuzzy biclosure spaces has also been studied.

KEYWORDS : Fuzzy closure space, Fuzzybiclosure space, Fuzzy biclosed map,Pairwise fuzzy biclosed map.

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1. INTRODUCTION

The concept of closure spaces were introduced by E.Čech[1] . Closure Space is a generalization of topological spaces and today,the theory of closure space is one of the most popular theory of mathematics which finds many interesting applications in various areas.

Later on C.Boonpok[2] introduced the notion of biclosure spaces. Such spaces are equipped with two arbitrary closure operators. In 1985, A.S. Mashhour and M.H Ghanim[3] extended the concept of closure space to fuzzy sets and laid the foundation of fuzzy closure space.

Recently, Tapi and Navalakhe[4,5] have introduced and studied the concept of fuzzy biclosure spaces. In this paper the conceptof fuzzy biclosed (fuzzy biopen) maps and pairwise fuzzy biclosed (pairwise fuzzy biopen) maps in fuzzy biclosure spaces has been introduced and study their properties.

2. PRELIMINARIES

Definition 2.1[4]. A *fuzzy biclosure space* is a triple (X,u_1,u_2) where X is a non-empty set and u_1,u_2 are two fuzzy closure operators on X which satisfy the following properties:

(i) $u_1(0_X)=0_X$ and $u_2(0_X)=0_X$

(ii) $\mu \le u_1\mu$ and $\mu \le u_2\mu$ for all $\mu \le I^X$

(iii) $u_1(\mu \vee \upsilon)=u_1\mu \vee u_1\upsilon$ and $u_2(\mu \vee \upsilon)=u_2\mu \vee u_2\upsilon$ for all $\mu,\upsilon \le I^X$.

Definition 2.2[4]. A subset μ of a fuzzy biclosure space (X,u_1,u_2) is called fuzzy closed if $u_1u_2\mu=\mu$. The complement of fuzzy closed set is called fuzzy open.

Definition 2.3[4]. Let (X,u_1,u_2) be a fuzzy biclosure space. A fuzzy biclosure space (Y,v_1,v_2) is called a subspace of (X,u_1,u_2) if $Y \le X$ and $v_i\mu=u_i\mu \wedge 1_Y$ for each $i\in\{1,2\}$ and each subset $\mu \le Y$. If 1_Y is fuzzy closed in (X,u_1) and (X,u_2) then the fuzzy subspace (Y,v_1,v_2) is also said to be fuzzy closed.

Definition 2.4[5]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is called fuzzy continuous if $f^{-1}(\mu)$ is a fuzzy closed subset of

(X,u_1,u_2) for every fuzzy closed subset μ of (Y,v_1,v_2).

Clearly, it is easy to prove that a map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is fuzzy continuous if and only if $f^{-1}(\upsilon)$ is a fuzzy open subset of (X,u_1,u_2) for every fuzzy open subset υ of (Y,v_1,v_2).

Definition2.5[5]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is said to be fuzzy closed (resp. fuzzy open) if $f(\mu)$ is fuzzy closed (resp. fuzzy open) subset of (Y,v_1,v_2) whenever μ is a fuzzy closed (resp. fuzzy open) subset of (X,u_1,u_2).

Definition 2.6 [5]. The product of a family $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ of fuzzy biclosure spaces denoted by $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$ is the fuzzy biclosure space $\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$ where $\left(\prod_{\alpha\in J}X_\alpha,u^i\right)$ for $i\in\{1,2\}$ is the product of the family of fuzzy closure spaces $\{X_\alpha,u^i:\alpha\in J\}$.

Remark 2.7. Let $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)=\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$.Then for each $\mu\le\prod_{\alpha\in J}X_\alpha$, $u^1u^2\mu=\prod_{\alpha\in J}u_\alpha^1u_\alpha^2\pi_\alpha(\mu)$.

Proposition 2.8[5]. Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces and let $\beta\in J$.

Then $\eta\le X_\beta$ is a fuzzy closed subset of $(X_\beta,u_\beta^1,u_\beta^2)$ if and only if $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$.

Proposition 2.9. Let $\{(X_\alpha,u_\alpha{}^1,u_\alpha{}^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces. Then for each $\beta\in J$, the projection map $\pi_\beta:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to(X_\beta,u_\beta^1,u_\beta^2)$ is fuzzy closed.

Proof. Let η be a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_\alpha, u_\alpha^1, u_\alpha^2\right)$. Then η is a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_\alpha, u_\alpha^1\right)$ and $\prod_{\alpha\in J}\left(X_\alpha, u_\alpha^2\right)$ respectively. Since $\pi_\beta : \prod_{\alpha\in I}\left(X_\alpha, u_\alpha^1\right) \to \left(X_\beta, u_\beta^1\right)$ is fuzzy closed, $\pi_\beta(\eta)$ is a fuzzy closed subset of $\left(X_\beta, u_\beta^1\right)$. Similarly, since $\pi_\beta : \prod_{\alpha\in I}\left(X_\alpha, u_\alpha^2\right) \to \left(X_\beta, u_\beta^2\right)$ is fuzzy closed, $\pi_\beta(\eta)$ is a fuzzy closed subset of $\left(X_\beta, u_\beta^2\right)$. Consequently, $\pi_\beta(\eta)$ is a fuzzy closed subset of $\left(X_\beta, u_\beta^1, u_\beta^2\right)$. Hence, the map π_β is fuzzy closed.

3. FUZZY BICLOSED MAPS

Definition 3.1. Let (X, u_1, u_2) and (Y, v_1, v_2) be fuzzy biclosure spaces and let $i \in \{1,2\}$. A map $f:(X, u_1, u_2) \to (Y, v_1, v_2)$ is called i- fuzzyclosed (resp. i- fuzzy open)if the map $f:(X, u_i) \to (Y, v_i)$ is fuzzy closed (resp. fuzzy open). A map f is called fuzzy closed (resp. fuzzy open)if f is i- fuzzy closed (resp. i- fuzzy open) for each $i \in \{1,2\}$.

Definition 3.2. Let (X, u_1, u_2) and (Y, v_1, v_2) be fuzzy biclosure spaces. A map $f:(X, u_1, u_2) \to (Y, v_1, v_2)$ is called fuzzy biclosed (resp. fuzzy biopen) if the map $f:(X, u_1) \to (Y, v_2)$ is fuzzy closed (resp. fuzzy open).

Proposition 3.3. Let (X, u_1, u_2), (Y, v_1, v_2) and (Z, w_1, w_2) be fuzzybiclosure spaces. If themap $f:(X, u_1, u_2) \to (Y, v_1, v_2)$ is 1-fuzzy closed and the map $h:(Y, v_1, v_2) \to (Z, w_1, w_2)$ is fuzzy biclosed, then the map $h \circ f:(X, u_1, u_2) \to (Z, w_1, w_2)$ is fuzzy biclosed.

Proof. Let η be a fuzzy closed subset of (X, u_1). Since the map f is 1-fuzzy closed, $f(\eta)$ is a fuzzy closed subset of (Y, v_1). Since the map h is fuzzy biclosed, $h(f(\eta))$ is a fuzzy closed subset of (Z,w_2). Hence, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2). Consequently, the map $h\circ f$ is fuzzy biclosed.

Proposition 3.4. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. Let $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ be maps. Then

(i) If $h\circ f$ is fuzzy biclosed and f is surjective and 1- fuzzy continuous, then h is fuzzy biclosed.

(ii) If $h\circ f$ is fuzzy biclosed and h is injective and 2- fuzzy continuous, then f is fuzzy biclosed.

Proof.(i) Let η be a fuzzy closed subset of (Y,v_1). Since the map f is 1-fuzzy continuous, $f^{-1}(\eta)$ is a fuzzy closed subset of (X,u_1). Since $h\circ f$ is fuzzy biclosed and the map f is surjective, $h\circ f\left(f^{-1}(\eta)\right)=h(\eta)$ is a fuzzy closed subset of (Z,w_2). Hence, the map h is fuzzy biclosed.

(ii) Let η be a fuzzy closed subset of (X,u_1). Since the map $h\circ f$ is fuzzy biclosed, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2). Since h is 2-fuzzy continuous and injective, $h^{-1}(h\circ f(\eta))=f(\eta)$ is a fuzzy closed subset of (Y,v_2). Therefore, the map f is fuzzy biclosed.

The following statement is evident.

Proposition 3.5. Let $\left\{\left(X_\alpha,u_\alpha^1,u_\alpha^2:\alpha\in J\right)\right\}$ and $\left\{\left(Y_\alpha,v_\alpha^1,v_\alpha^2\right):\alpha\in J\right\}$ be families of fuzzy biclosure spaces. For each $\alpha\in J$, let $f_\alpha:\left(X_\alpha,u_\alpha^1,u_\alpha^2\right)\to\left(Y_\alpha,v_\alpha^1,v_\alpha^2\right)$ be a surjection and let $f:\prod_{\alpha\in J}\left(X_\alpha,u_\alpha^1,u_\alpha^2\right)\to\prod_{\alpha\in J}\left(Y_\alpha,v_\alpha^1,v_\alpha^2\right)$ be defined by $f((x_\alpha))_{\alpha\in I}=(f_\alpha(x_\alpha))_{\alpha\in I}$. Then the map f is fuzzy biclosed if and only if the map f_α is fuzzy biclosed for each $\alpha\in J$.

Proof. Let $\beta\in J$ and let η be a fuzzy closed subset of $\left(X_\beta,u_\beta^1\right)$. Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_\alpha,u_\alpha^1\right)$. Since the map f is fuzzy biclosed, $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha, v_\alpha^2)$. But $f\left(\eta \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in J}} X_\alpha\right) = f_\beta(\eta) \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in J}} Y_\alpha,$ hence $f_\beta(\eta) \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in J}} Y_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha, v_\alpha^2)$. By Proposition 2.8, $f_\beta(\eta)$ is a fuzzy closed subset of (Y_β, v_β^2). Hence, the map f_β is fuzzy biclosed.

Conversely, let the map f_β be fuzzy biclosed for each $\beta\in J$. Suppose that the map f is not fuzzy biclosed. Then there exists a fuzzy closed subset η of $\prod_{\alpha\in J}(X_\alpha, u_\alpha^1)$ such that $\prod_{\beta\in J} v_\beta^2 \pi_\beta(f(\eta)) \not\leq f(\eta)$. Therefore, there exists $\beta\in J$ such that $v_\beta^2 f_\beta(\pi_\beta(\eta)) \not\leq f_\beta(\pi_\beta(\eta))$. But $\pi_\beta(\eta)$ is a fuzzy closed subset of (X_β, u_β^1) and f_β is fuzzy biclosed, $f_\beta(\pi_\beta(\eta))$ is a fuzzy closed subset of (Y_β, v_β^2). This is a contradiction. Therefore, the map f is fuzzy biclosed.

4. PAIRWISE FUZZY BICLOSED MAP

The purpose of this section is to introduce the concept of pairwise fuzzy biclosed (pairwise fuzzy biopen) maps in fuzzy biclosure spaces and study some oftheir properties.

Definition 4.1. Let (X, u_1, u_2) and (Y, v_1, v_2) be fuzzy biclosure spaces. A map $f:(X, u_1, u_2)\to(Y, v_1, v_2)$ is called pairwise fuzzy *biclosed* (resp. pairwise fuzzy bi*open*)if the map $f:(X, u_1)\to(Y, v_2)$ and $f:(X, u_2)\to(Y, v_1)$ are fuzzy *closed* (resp. fuzzy *open*)

Remark 4.2. Every pairwise fuzzy biclosed map is fuzzy biclosed and every pairwise fuzzy biopen map is fuzzy biopen.

Proposition 4.3. Let (X, u_1, u_2), (Y, v_1, v_2) and (Z, w_1, w_2) be fuzzy biclosure spaces. If the map $f:(X, u_1, u_2)\to(Y, v_1, v_2)$ is fuzzy closed and $h:(Y, v_1, v_2)\to(Z, w_1, w_2)$ is pairwise fuzzy biclosed, then the map $h\circ f:(X, u_1, u_2)\to(Z, w_1, w_2)$ is pairwise fuzzy biclosed.

Proof. Let η be a fuzzy closed subset of (X, u_1) and let γ be a fuzzyclosed subset of (X, u_2). Since the map f is fuzzy closed, $f(\eta)$ is a fuzzy closed subset of (Y, v_1) and $f(\gamma)$ is a fuzzy closed subset of (Y,v_2). Since the map h is pairwise fuzzy biclosed, $h(f(\eta))$ is a fuzzy closed subset of (Z,w_2) and $h(f(\gamma))$ is a fuzzy closed subset of (Z,w_1). Hence, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2) and $h\circ f(\gamma)$ is a fuzzy closed subset of (Z,w_1). Consequently, the map $h\circ f$ is pairwise fuzzy biclosed.

Proposition 4.4. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. If the map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is fuzzy open and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ is pairwise fuzzy biopen, then the map $h\circ f:(X,u_1,u_2)\to(Z,w_1,w_2)$ is pairwise fuzzy biopen.

Proof. It is obvious

Proposition 4.5. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. Let $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ be maps. Then

- **(i)** If $h\circ f$ is pairwise fuzzy biclosed and f is surjective and fuzzy continuous, then h is pairwise fuzzy biclosed.
- **(ii)** If $h\circ f$ is pairwise fuzzy biclosed and h is injective and fuzzy continuous, then f is pairwise fuzzy biclosed.

Proof. **(i)**Let η be a fuzzy closed subset of (Y,v_1) and let γ be a fuzzy closed subset of (Y,v_2). Since the map f is fuzzy continuous, $f^{-1}(\eta)$ is a fuzzy closed subset of (X,u_1) and $f^{-1}(\gamma)$ is a fuzzy closed subset of (X,u_2). Since the map $h\circ f$ is pairwise fuzzy biclosed and f is surjective, $h\circ f(f^{-1}(\eta))=h(\eta)$ is a fuzzy closed subset of (Z,w_2) and $h\circ f(f^{-1}(\gamma))=h(\gamma)$ is a fuzzy closed subset of (Z,w_1). Hence, the map h is pairwise fuzzy biclosed.

(ii) Let η be a fuzzy closed subset of (X,u_1) and let γ be a fuzzy closed subset of (X,u_2). Since the map $h\circ f$ is pairwise fuzzy biclosed, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2) and $h\circ f(\gamma)$ is a fuzzy closed subset of (Z,w_1). Since the map h is fuzzy continuous and injective

$h^{-1}(h\circ f(\eta))=f(\eta)$ is a fuzzy closed subset of (Y,v_2) and $h^{-1}(h\circ f(\gamma))=f(\gamma)$ is a fuzzy closed subset of (Y,v_1). Therefore, the map f is pairwise fuzzy biclosed.

Proposition 4.6. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. Let $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ be maps. Then

(i) If $h\circ f$ is pairwise fuzzy biopen and f is surjective and fuzzy continuous, then h is pairwise fuzzy biopen.

(ii) If $h\circ f$ is pairwise fuzzy biopen and h is injective and fuzzy continuous, then f is pairwise fuzzy biopen.

Proof. It is obvious.

Proposition 4.7. Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ and $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ be families of fuzzy biclosure spaces. For each $\alpha\in J$, let $f_\alpha:X_\alpha\to Y_\alpha$ be a surjection and let $f:\prod_{\alpha\in J}X_\alpha\to\prod_{\alpha\in J}Y_\alpha$ be defined by $f((x_\alpha)_{\alpha\in I})=(f_\alpha(x_\alpha))_{\alpha\in I}$. Then the map $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ is pairwise fuzzy biclosed if and only if the map $f_\alpha:(X_\alpha,u_\alpha^1,u_\alpha^2)\to(Y_\alpha,v_\alpha^1,v_\alpha^2)$ is pairwise fuzzy biclosed for each $\alpha\in J$.

Proof. Let $\beta\in J$ and let η be a fuzzy closed subset of (X_β,u_β^1) and γ bea fuzzy closed subset of (X_β,u_β^2). Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)$ and $\gamma\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^2)$. Since f is pairwise fuzzy biclosed, $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^2)$ and $f\left(\gamma\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1)$. But

$f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)=f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$ and $f\left(\gamma\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)=f_\beta(\gamma)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$, hence $f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha{}^2)$ and $f_\beta(\gamma)\times\prod_{\alpha\in J}Y_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1)$. By Proposition 2.8, $f_\beta(\eta)$ is a fuzzy closed subset of (Y_β,v_β^2) and $f_\beta(\gamma)$ is a fuzzy closed subset of (Y_β,v_β^1). Hence, f_β is pairwise fuzzy biclosed.

Conversely, let the map f_β be pairwise fuzzy biclosed for each $\beta\in J$. Suppose that the map f is not pairwise fuzzy biclosed. Then $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^2)$ is not fuzzy closed or $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1)$ is not fuzzy closed. If $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^2)$ is not fuzzy closed. Then there exists a fuzzy closed subset η of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)$ such that $\prod_{\beta\in J}v_\beta^2\pi_\beta(f(\eta))\not\leq f(\eta)$. Therefore, there exists $\beta\in J$ such that $v_\beta^2 f_\beta(\pi_\beta(\eta))\not\leq f_\beta(\pi_\beta(\eta))$. By Lemma 2.9, $\pi_\beta(\eta)$ is a fuzzy closed subset of (X_β,u_β^1). Since map f_β is pair wise fuzzy biclosed, $f_\beta(\pi_\beta(\eta))$ is a fuzzy closed subset of (Y_β,v_β^2). This is a contradiction. If $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1)$ is not fuzzy closed. Then there exists a fuzzy closed subset γ of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^2)$ such that $\prod_{\beta\in J}v_\beta^1\pi_\beta(f(\gamma))\not\leq f(\gamma)$. Therefore, there exists $\beta\in J$ such that $v_\beta^1 f_\beta(\pi_\beta(\gamma))\not\leq f_\beta(\pi_\beta(\gamma))$. By Lemma 2.9, $\pi_\beta(\gamma)$ is a fuzzy closed subset of (X_β,u_β^2). Since f_β is pairwise fuzzy biclosed, $f_\beta(\pi_\beta(\gamma))$ is a fuzzy closed subset of (Y_β,v_β^1). This is a contradiction. Therefore, the map f is pairwise fuzzy biclosed.

Proposition 4.8. Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ and $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ be families of fuzzy biclosure spaces. For each $\alpha\in J$, let $f_\alpha:X_\alpha\to Y_\alpha$ be a surjection and let $f:\prod_{\alpha\in J}X_\alpha\to\prod_{\alpha\in J}Y_\alpha$ be defined by

$f\left((x_\alpha)_{\alpha\in J}\right) = \left(f_\alpha(x_\alpha)\right)_{\alpha\in J}$. If the map $f:\prod_{\alpha\in J}\left(X_\alpha,u_\alpha^1,u_\alpha^2\right)\rightarrow\prod_{\alpha\in J}\left(Y_\alpha,v_\alpha^1,v_\alpha^2\right)$ is pairwise fuzzy biopen, then the map $f_\alpha:\left(X_\alpha,u_\alpha^1,u_\alpha^2\right)\rightarrow\left(Y_\alpha,v_\alpha^1,v_\alpha^2\right)$ is pairwise fuzzy biopen for each $\alpha\in J$.

Proof. Obvious.

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