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The Rank of A Fuzzy Magic Graphs With Adjacency Matrix

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ABSTRACT

The definition of adjacency matrix of a graph is observed. The matrix representation of a graph leads very new approach in computer science, engineering and network problems. In this paper we find out the rank of some fuzzy magic graphs such as cycle ,path and star graphs.

KEYWORDS: Fuzzy graph, Fuzzy magic graph, Adjacency matrix, Cycle, Star, Path.

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INTRODUCTION:

Fuzzy set is a newly emerging mathematical frame work exemplify the phenomenon of uncertainly in real life tribulations[1]. A fuzzy set is defined mathematically by assigning a value to each possible individual in the universe of discourse, representing its grade of membership which corresponds to the degree to which that individually similar or compatible with the concept represented the fuzzy set .Based on Zadeh's Fuzzy relation the definition of a fuzzy graph was introduced by Kauffmann in 1973.One of the first important papers on fuzzy graph theory was by Azriel Rosenfeld[5]. Rosenfeld introduce and examined such concepts as paths, connectedness , bridges , cut vertices , forests and trees[2].A fuzzy graph is the generalization of the crisp graph. Therefore it is natural many properties are similar to crisp graph and also it deviates at many places. The notation of magic graph was first introduced by Sunitha and Vijayakumar in 1964[4].

In mathematics and computer science, an adjacency matrix is a means of representing which vertices of a graph are adjacent to which other vertices. The adjacency matrix A of graph G does depend on the ordering of the nodes of G, that is, a different ordering of nodes may result in a different adjacency matrix[3]. In section 1, we discussed basic definitions and in section 2, the rank of fuzzy magic graphs such as cycle, path, star graphs are introduced.

PRELIMINARIES:

DEFINITION 2.1

A fuzzy graph (undirected and without loops) $G = (V, \sigma, \mu)$ is a non empty finite set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ with $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$, for all $u,v \in V$,where V is a finite nonempty set and $\wedge$ denote minimum.

DEFINITION 2.2

A fuzzy graph $G=(V, \sigma, \mu)$ is called a fuzzy magic graph if there are two bijective functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ with restricted the conditions $\mu(u,v) < \sigma(u) + \sigma(v)$ and $\sigma(u)+\mu(uv)+\sigma(v) = \lambda(G) \leq 1$ where , $\lambda(G)$ is a real constant for all $u,v \in G$.

DEFINITION 2.3

A path P in a fuzzy graph is a sequence of distinct vertices $v_1, v_2, v_3, \ldots, v_n$ such that $\mu(v_i, v_{i+1}) > 0$;$1 \leq i \leq n$; here $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edge of the path.

DEFINITION 2.4

A cycle is called a fuzzy cycle if it contains more than one weakest arc.

DEFINITION 2.5

A fuzzy graph is called a fuzzy star graph, if there are two vertex sets X and Y with $|X|=1$ and $|Y| > 1$,such that $\mu(y,x_i) > 0$ and $\mu(x_i,x_{j+1}) = 0$, $1 \leq i \leq n$ and denoted by $S_{1,n}$.

RANK OF MATRIX

The rank of a matrix is defined as the maximum number of linearly independent column vectors in the matrix or the maximum number of linearly independent row vectors in the matrix.

ADJACENCY MATRIX OF A FUZZY MAGIC GRAPH

If is a fuzzy magic graph then its adjacency matrix is defined as X where $X_{i,j} = \mu(v_i,v_j)$ for $i \neq j$; and when i=j,

$$X_{i,i} = \sigma(v_i) \quad \text{if the fuzzy relation is reflexive}$$
$$= 0 \quad \text{if the fuzzy relation is not reflexive}$$

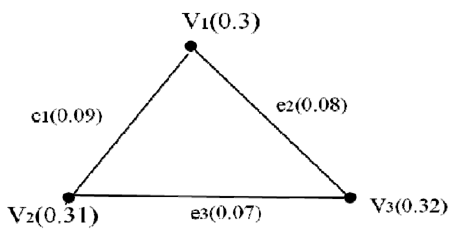
In this section let us consider the fuzzy magic graphs with their fuzzy relations being not reflexive.

MAIN RESULTS:

ADJACENCY MATRIX OF SOME FUZZY MAGIC GRAPHS:

Now we consider some C_n graphs to find some results on the rank of the adjacency matrix for C_n graph.

1. C_3 graph:



The adjacency matrix is

$$A(C_3) = \begin{bmatrix} 0 & 0.09 & 0.08 \\ 0.09 & 0 & 0.07 \\ 0.08 & 0.07 & 0 \end{bmatrix}$$

Now ,rank(ρ (C_3) = 3

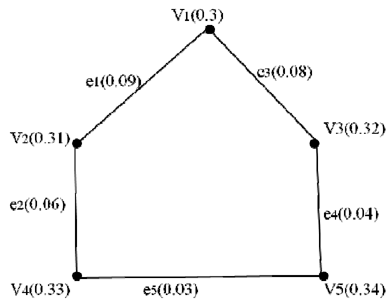
2. C_4 graph:

The adjacency matrix is

$$A(C_4) = \begin{bmatrix} 0 & 0.09 & 0.08 & 0 \\ 0.09 & 0 & 0 & 0.06 \\ 0.08 & 0 & 0 & 0.05 \\ 0 & 0.06 & 0.05 & 0 \end{bmatrix}$$

Now ,rank(ρ (C_4) = 4

3.C_5 graph:

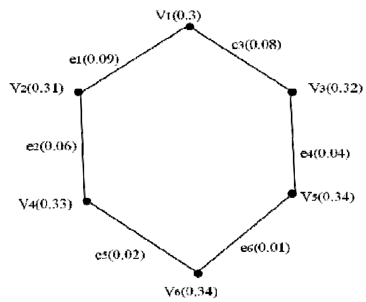


The adjacency matrix is

$$A(C_5) = \begin{bmatrix} 0 & 0.09 & 0.08 & 0 & 0 \\ 0.09 & 0 & 0 & 0.06 & 0 \\ 0.08 & 0 & 0 & 0 & 0.04 \\ 0 & 0.06 & 0 & 0 & 0.03 \\ 0 & 0 & 0.04 & 0.03 & 0 \end{bmatrix}$$

Now ,rank(ρ (C_5) = 5

4. C_6 graph:

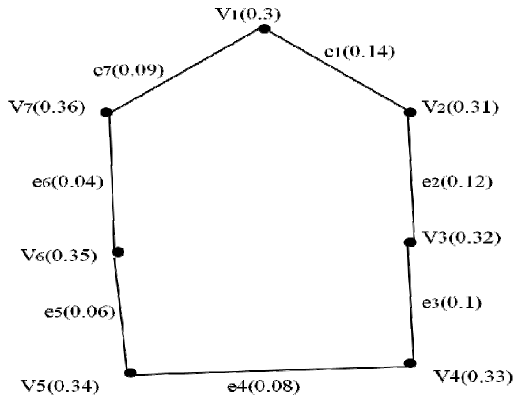


The adjacency matrix is

$$A(C_6) = \begin{bmatrix} 0.09 & 0 & 0.08 & 0 & 0 & 0 \\ 0.09 & 0 & 0 & 0.06 & 0 & 0 \\ 0.08 & 0 & 0 & 0.04 & 0 & 0 \\ 0 & 0.06 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 0.04 & 0 & 0 & 0.01 \\ 0 & 0 & 0 & 0.02 & 0.01 & 0 \end{bmatrix}$$

Now ,rank(ρ (C_6) = 6

5. C_7 graph:



The adjacency matrix is

$$\begin{bmatrix} 0 & 0.14 & 0 & 0 & 0 & 0 & 0.09 \\ 0.14 & 0 & 0.12 & 0 & 0 & 0 & 0 \\ 0 & 0.12 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0.08 & 0 & 0 \\ 0 & 0 & 0 & 0.08 & 0 & 0.06 & 0 \\ 0 & 0 & 0 & 0 & 0.06 & 0 & 0.04 \\ 0.09 & 0 & 0 & 0 & 0 & 0.04 & 0 \end{bmatrix}$$

Now ,rank(ρ (C_7) = 7

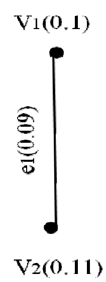
On the basis of above observations, for n $\geq$ 3 ,we may generalize as follows:

RESULTS ON THE ADJACENCY MATRIX OF THE C_n GRAPHS

If A is the adjacency matrix of the C_n graph on n (n $\geq$ 3) vertices then the rank A is ρ (A) = n , if n is odd /even

Now we consider some $S_{1,n}$ graphs to find some results on the rank of the adjacency matrix for $S_{1,n}$ graph.

1 .$S_{1,1}$ graph

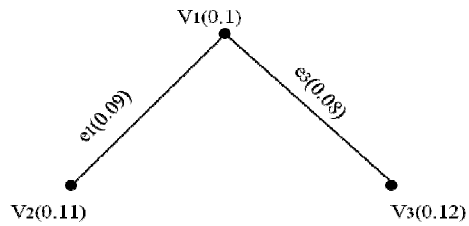


The adjacency matrix is

$$A(S_{1,1}) = \begin{bmatrix} 0 & 0.09 \\ 0.09 & 0 \end{bmatrix}$$

Now ,rank(ρ ($S_{1,1}$) = 2

1 .$S_{1,2}$ graph

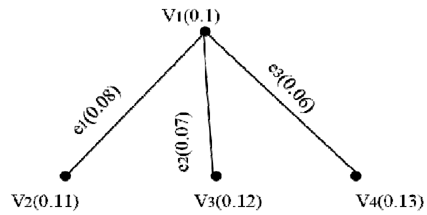


The adjacency matrix is

$$A(S_{1,2}) = \begin{bmatrix} 0 & 0.09 & 0.08 \\ 0.09 & 0 & 0 \\ 0.08 & 0 & 0 \end{bmatrix}$$

Now ,rank(ρ $(S_{1,2})$ = 2

3 .$S_{1,3}$ graph

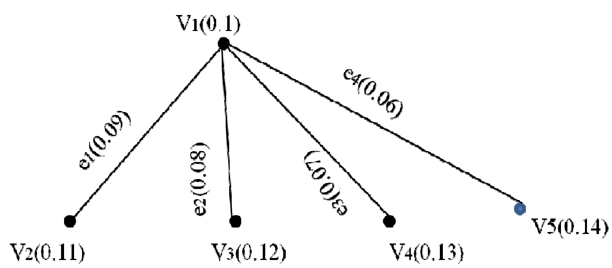


The adjacency matrix is

$$A(S_{1,3}) = \begin{bmatrix} 0 & 0.08 & 0.07 & 0.06 \\ 0.08 & 0 & 0 & 0 \\ 0.07 & 0 & 0 & 0 \\ 0.06 & 0 & 0 & 0 \end{bmatrix}$$

Now ,rank(ρ $(S_{1,3})$ = 2

4 .$S_{1,4}$ graph

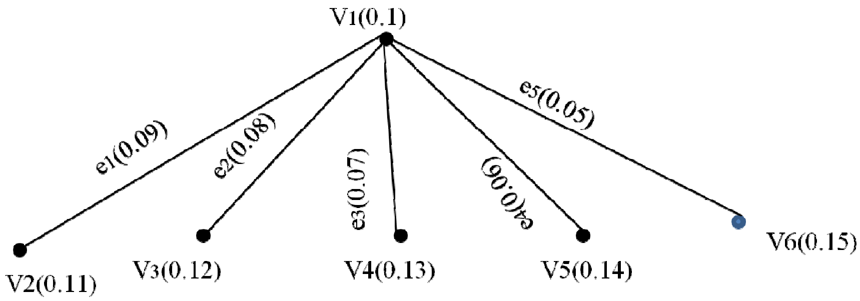


Te adjacency matrix is

$$A(S_{1,4}) = \begin{bmatrix} 0 & 0.09 & 0.08 & 0.07 & 0.06 \\ 0.09 & 0 & 0 & 0 & 0 \\ 0.08 & 0 & 0 & 0 & 0 \\ 0.07 & 0 & 0 & 0 & 0 \\ 0.06 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now ,rank(ρ $(S_{1,4})$ = 2

5 .$S_{1,5}$ graph



The adjacency matrix is

$$A(S_{1,5}) = \begin{bmatrix} 0 & 0.09 & 0.08 & 0.07 & 0.06 & 0.05 \\ 0.09 & 0 & 0 & 0 & 0 & 0 \\ 0.08 & 0 & 0 & 0 & 0 & 0 \\ 0.07 & 0 & 0 & 0 & 0 & 0 \\ 0.06 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now ,rank(ρ ($S_{1,5}$) = 2

On the basis of above observations, for n $\geq$ 1,we may generalize as follows.

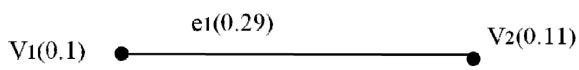
RESULTS ON THE ADJACENCY MATRIX OF THE $S_{1,n}$ GRAPH

If A is the adjacency matrix of the $\mathbf{S_{1,n}}$ graph on n (n $\geq$ 1) vertices then the rank A is

ρ (A) = 2 , if n is odd /even

Now we consider some P_n graphs to find some results on the rank of the adjacency matrix for P_n graph.

1 .P_2 graph

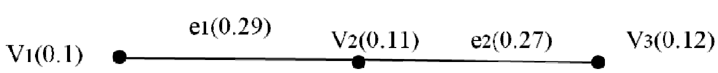


The adjacency matrix is

$$A(P_2)= \begin{bmatrix} 0 & 0.29 \\ 0.29 & 0 \end{bmatrix}$$

Now ,rank(ρ (P_2) = 2

2.P_3 graph

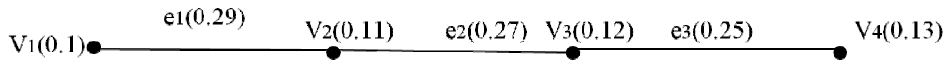


The adjacency matrix is

$$A(P_3) = \begin{bmatrix} 0 & 0.29 & 0 \\ 0.29 & 0 & 0.27 \\ 0 & 0.27 & 0 \end{bmatrix}$$

Now ,rank(ρ (P_3) = 3

3.P_4 graph

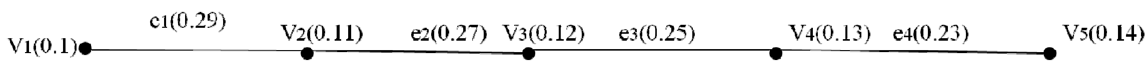


The adjacency matrix is

$$A(P_4) = \begin{bmatrix} 0 & 0.29 & 0 & 0 \\ 0.29 & 0 & 0.27 & 0 \\ 0 & 0.27 & 0 & 0.25 \\ 0 & 0 & 0.25 & 0 \end{bmatrix}$$

Now ,rank(ρ (P_4) = 4

3. P_5 graph



The adjacency matrix is

$$A(P_5) = \begin{bmatrix} 0 & 0.29 & 0 & 0 & 0 \\ 0.29 & 0 & 0.27 & 0 & 0 \\ 0 & 0.27 & 0 & 0.25 & 0 \\ 0 & 0 & 0.25 & 0 & 0.23 \\ 0 & 0 & 0 & 0.23 & 0 \end{bmatrix}$$

Now ,rank(ρ (P_5) = 5

On the basis of above observations, for $n \geq 2$,we may generalize as follows:

RESULTS ON THE ADJACENCY MATRIX OF THE P_n GRAPH

If A is the adjacency matrix of the P_n graph on n ($n \geq 2$) vertices then the rank A is

ρ(A) = n , if n is odd /even

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