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A study onharmonic mean labeling of joins of kite graph

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ABSTRACT

Let G= (V, E) be a graph with p vertices and q edges. A graph G={V, E} with p vertices and q edges is said to be a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2, … q+1 in such a way that when each edge e=uv is labelled with $f(e=uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct. In this case f is called Harmonic mean labelling of G. In this paper we have identified (m,n) Kite graph and attached an edge to form a join to the kite graph and proved that the joins of (m,n) kite graphs is harmonic mean labelling and also have exhibited some important results connecting the joins of a (m,n) kite graph.

KEY WORDS: (m,n) Kite Graph, Harmonic Mean labelling , Harmonic Mean graph, Joins of (m,n) Kite graphs

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1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian J. A[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa.The vertex set is denoted by V(G) and the edge set is denoted by E(G). Mean Labeling of graphs is introduced by .Somasundaran S and .Ponraj R [2] and Sandhya S .S and Somasundaram S [3] introduced Harmonic mean labelling of graphs. Motivated toward the labelling of Harmonic mean labelling of graphs we have identified the (m,n) Kite graph and tried to attach edge to the (m,n) Kite graphs to form a chain of (m,n) Kite graphs which we call as Join of (m,n) Kite Graph. In this paper we intend to prove that the Join of Kite Graph is Harmonic Mean labelling graph and we exhibit some important results connecting the joins of a (m,n) kite graph.

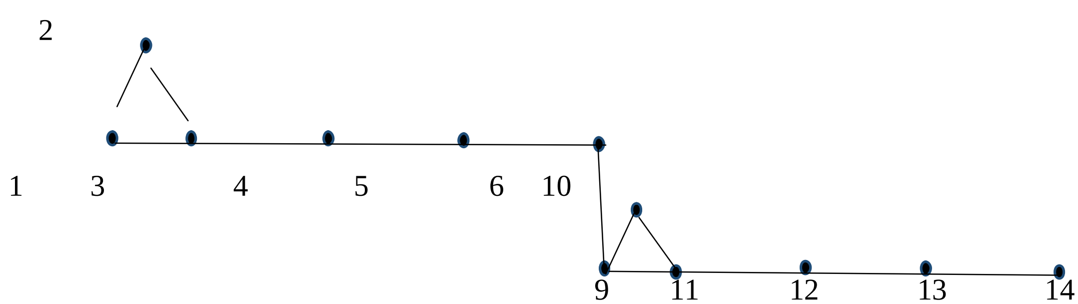
2. PRELIMINARIES

Definition 2.1: A graph G with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2, … q+1 in such a way that when each edge e=uv is labelled with $f(e=uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct.

Definition .2.2: An (m,n) Kite graph consists of cycle of length m with n edges path attached to one vertex of a cycle

Definition .2.3 : For a (m,n) Kite graph we attach an edge from the Kite graph to another (m,n) Kite graph and we call the same as 1-Join of (m,n) Kite graph.

The 1-Join of (3,3) Kite graph is given as



The above is a 1-join of (3,3) Kite graph with a cycle of length 3 and with 3 edges path attached to one vertex of a cycle is joined by an edge with another (3,3) Kite graph with a cycle of length 3 and with 3 edges path attached to one vertex of a cycle

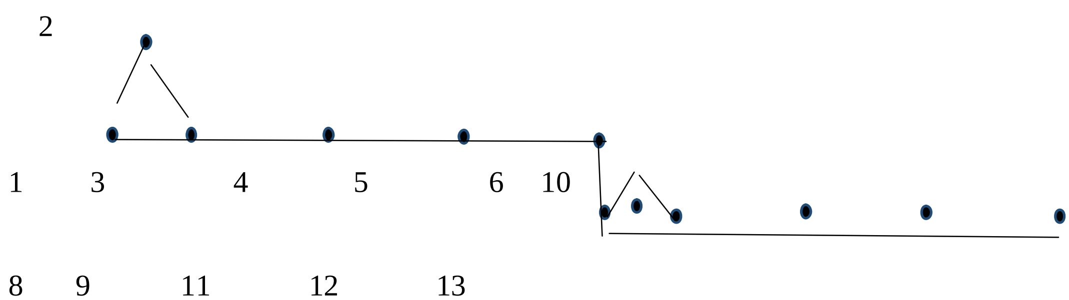
Similarly we can construct 2-join (m,n) Kite graph by attaching one more edge with another (3,3) kite graph . Continuing this process we get a chain of Join of (m,n) Kite graph.

Note: We here exclusively study the case of (m_1, n_1) Kite graph attached by an edge e_1 to (m_2, n_2) Kite graph , (m_2, n_2) Kite graph attached by an edge e_2 to (m_3, n_3) Kite graph and so on (m_{n-1}, n_{n-1}) Kite graph attached by an edge e_n to (m_n, n_n) Kite graph respectively such that $m_1 = m_2 = m_3 = ... = m_n$ and $n_1 = n_2 = n_3 = ... = n_n$ where each m_i is the cycle of same length and n_i is the number of edges in the path of same length.

We can discuss the varieties of Join of (m,n) Kite graph as follows

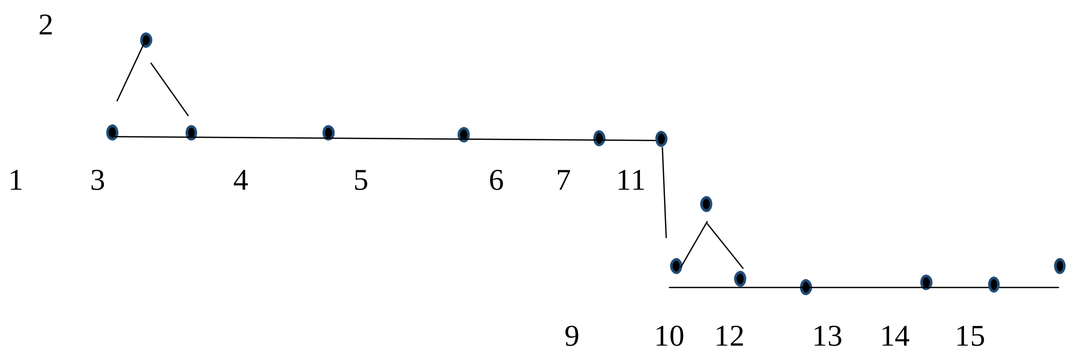
Case.1 When m=n

Example : 1-Join (3,3) Kite Graph with (3,3) Kite graph



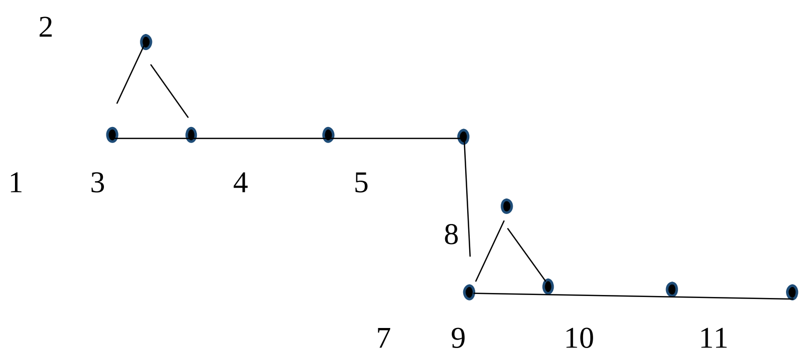
Case.2 When m<n

Example : 1-Join (3,4) Kite Graph with (3,4) Kite graph



Case.3 When m>n

Example : 1-Join (3,2) Kite Graph with (3,2) Kite graph



Note : In general we can consider any form of the above mentioned cases of (m,n) Kite graph to join by an edge to form a chain of (m,n) Kite graphs and let us consider 1-Join of (m,n) Kite graph as the basic graph and show that in the following theorem is a Harmonic Mean graph.

3. MAIN RESULTS

Theorem .3.1: The 1- Join of (m,n) Kite graph G is a Harmonic Mean graph.

Proof: Let G=1-Join of (m,n) Kite graph

Let us prove that G is a Harmonic Mean graph

Let us prove the theorem by labelling the vertices of the graph for all the three cases as mentioned above

We have the number of vertices in (m,n) Kite graph as m+n and the number of edges in (m,n) Kite graph as m+n. Now adding one edge between the (m,n) kite graph and another (m,n) kite graph we have the number of vertices in 1-Join of (m,n) Kite graph is 2(m+n) and the number of edges in 1-Join of (m,n) Kite graph is 2(m+n)+1.

Now we follow the following Scheme for labelling the vertices of (m,n) Kite graph with another (m,n) kite graph

Total Labels required to label the vertices of 1-Join (m,n) Kite graph the is 2(m+n) and we label them as 1,2,3…2(m+n) .

Let us denote the Vertex Set of first (m,n) Kite graph as $V=\{u_1,u_2,u_3,u_4,...u_{m+n}\}$ and the vertex set of second (m,n) Kite graph as $V^1=\{u^1{}_1,u^1{}_2,u^1{}_3,u^1{}_4...u^1{}_{m+n}\}$. The edge set of first (m,n) Kite graph is $E=\{e_1,e_2,e_3,e_4,...e_{m+n}\}$ and the edge of the second (m,n) Kite Graph is $E^1=\{e^1{}_1,e^1{}_2,e^1{}_3,e^1{}_4...e^1{}_{m+n}\}$. Now by adding one edge between the first and second (m,n) Kite graph we have the Vertex set of 1-Join (m,n) Kite graph as $V(G)=\{u_1,u_2,u_3,u_4,...u_{m+n}\}\cup\{u^1{}_1,u^1{}_2,u^1{}_3,u^1{}_4...u^1{}_{m+n}\}$ and the edge set as $E(G)=\{e_1,e_2,e_3,e_4,...e_{m+n}\}\cup\{e^1{}_1,e^1{}_2,e^1{}_3,e^1{}_4...e^1{}_{m+n}\}\cup e^1$. The edges that are connecting between the first and second (m,n) graph is known .

Now let us label the vertices of first (m,n) kite graph in correspondence to the vertices of second (m,n) graph as follows

$f(u_i)=i$ for $1\le i\le m+n$

$f(u^1{}_i)=(m+n)+(i+1)$ for $1\le i\le m+n$

Now we can compute the induced edge labelling as follows

$$f^*(u_iu_{i+1})=\left\lceil\frac{2f(u_i)f(u_{i+1})}{f(u_i)+f(u_{i+1})}\right\rceil \text{or} \left\lfloor\frac{2f(u_i)f(u_{i+1})}{f(u_i)+f(u_{i+1})}\right\rfloor$$

$$f^{*}(u^{1}_{i}u^{1}_{i+1}) = \left\lceil \frac{2f(u^{1}_{i})f(u^{1}_{i+1})}{f(u^{1}_{i})+f(u^{1}_{i+1})} \right\rceil \text{ or } \left\lfloor \frac{2f(u^{1}_{i})f(u^{1}_{i+1})}{f(u^{1}_{i})+f(u^{1}_{i+1})} \right\rfloor$$

$$f^{*}(u_{m+n}u^{1}_{1}) = \left\lceil \frac{2f(u_{m+n})f(u^{1}_{1})}{f(u_{m+n})+f(u^{1}_{1})} \right\rceil \text{ or } \left\lfloor \frac{2f(u_{m+n})f(u^{1}_{1})}{f(u_{m+n})+f(u^{1}_{1})} \right\rfloor$$

Hence the induced edge labelling can be found to be distinct and hence it can be claimed that the given graph G is Harmonic mean labelling graph.

Hence the proof.

Theorem. 3.2: If graph G is a M-Join (m,n) Kite graph then Number of Vertices in M-Join (m,n) Kite graph = (M+1)(m+n) and Number of edges in M-Join (m,n) Kite graph = M(m+n+1)+ (m+n) where M is the number of Joins in (m,n) kite graph and m is the length of the cycle and n is the length of the path.

Proof: Let us prove the theorem by Method of Mathematical induction

Let us prove for the first positive integer i.e M=1

Given the graph G is 1- Join (m,n) Kite graph then according to the construction of the Join (m,n) Kite graph we have the number of vertices = 2(m+n) and the number of edges = 2(m+n)+1

Now let us have the following statement on the number of vertices and number of edges as

$P(V^{M}) = (M+1)(m+n)$

$P(E^{M}) = M(m+n+1)+(m+n)$

Now substituting M=1 we have the result true

Now let us assume the result is true for M=k

I.e $P(V^{k}) = (k+1)(m+n)$

$P(E^{k}) = k(m+n+1)+(m+n)$ is true

Now let us prove for M=k+1

i.e to prove $P(V^{k+1}) = (k+2)(m+n)$ and

$P(E^{k+1}) = (k+1)(m+n+1)+(m+n)$

Consider $P(V^{k+1}) = (k+1)(m+n)+(m+n)$

We know that each join increases by m+n vertices

Hence $P(V^{k+1}) = (k+1)(m+n)+(m+n)$

Similarly consider $P(E^{k+1}) = (k)(m+n+1)+(m+n)+(m+n+1)$

As we know that each join increases by m+n+1 edges

Hence $P(E^{k+1}) = (k+1)(m+n+1)+(m+n)$

Hence the proof of induction . Therefore the theorem is true for M-Joins of (m,n) Kite graph.

Definition.3.3 : Sum of the vertices of (m,n) Kite graph is defined as $S = \frac{(m+n)(m+n+1)}{2}$

Theorem 3.4: For any M-Join of (m,n) Kite Graph the Sum of the vertices of M-Join of (m,n) Kite graph is given by $S(M) = (2M+1)S$, Where S is the sum of (m,n) Kite graph

Proof: Let us prove the theorem by Method of Mathematical induction on the number of joins of M-Join of (m,n) Kite graph. Let us prove the result

$S(M) = (2M+1)S$ is true for M=1

Consider $S(M) = (2M+1)S$. Substituting M=1 we have $S(1) = (2(1)+1)S = 3S$ but

we know from the definition $S = \frac{(m+n)(m+n+1)}{2}$. Fix a graph 1-Join (3,3) Kite graph and we find that S=21 and hence S(1)= 63. Which is true. Similarly it is true for any value of m,n and hence the theorem is true for M=1.

Now let us assume that the theorem is true for M=K

i.e $S(K) = (2K+1)S$

Now let us prove that the theorem is true for M=K+1

i.e To prove $S(K+1) = (2(K+1)+1)S$

$S(K+1) = (2K+3)S$

Consider $S(K+1) = S(K) + 2S$, as each term increases by 2 S

Hence $S(K+1) = (2K+1)S + 2S$

On Simplifying we have

$S(K+1) = (2K+3)S$. Hence the theorem of induction and hence the result

$S(M) = (2M+1)S$ is true for all M, the number of joins.

Note :For instance Sum of the vertices of 1-Join of (m,n) kite graph is $S(1) = (2(1)+1)S = 3x21 = 63$.

Sum of the vertices of 2-Join of (m,n) kite graph is $S(2) = (2(2)+1)S = 5x21 = 105$ and so on. The above can be easily found out from the example.1 , example.2 and example.3 given for the three cases.

Theorem.3.5 : For any M-Join of (m,n) Kite Graph the sum of the vertices of M-Join of (m,n) Kite graph is given by $S(M) = S + \sum_{i=1}^{M} (m+n) d_i$ where d_i is the difference between the vertices

such that $d_1 = u_1^1 - u_1$, $d_i = u_1^{i+1} - u_1^i$, $2 \le i \le M-1$

Proof: Consider the M-Join of (m,n) Kite graph . Let us prove $S(M) = S + \sum_{i=1}^{M}(m+n)d_i$ by Method of Mathematical induction on the number of Joins M. Let us prove for one join M=1. For S(1)=63, the sum of the labels of the vertices of (3,3) Kite graph Similarly for (m,n) graph also it can be computed. R.H.S $S(M) = S + \sum_{i=1}^{M}(m+n)d_i$ of can be compute as

$S(1) = S + (m+n)d_1$, where $d_1 = u_1^1 - u_1$ which is 6 for (3,3) Kite graph similarly it can be computed for other (m,n) kite graphs.

Hence $S(1) = \frac{(m+n)(m+n+1)}{2} + (m+n)d_1$ which for (3,3) Kite graph is

$S(1) = 21 + 6x7 = 63$. Hence it is true for M=1 .

Now let us assume for the number of joins M=k

We assume that $S(k) = S + \sum_{i=1}^{k}(m+n)d_i$ is true.

To prove for M=k+1, To prove $S(k+1) = S + \sum_{i=1}^{k+1}(m+n)d_i$

Consider $S(k+1) = S(k) + (m+n)d_1 = S + \sum_{i=1}^{k}(m+n)d_i + (m+n)d_1$

Hence $S(k+1) = S + \sum_{i=1}^{k+1}(m+n)d_i$. Hence it is true for M=k+1 . Hence it is true for any M-join of (m,n) kite graph. Hence the theorem is proved.

RESULTS

In this paper we have considered M-Join of (m,n) Kite graph and proved that it is Harmonic mean labelling graph and have identified a generalisation method for the same.

CONCLUDING REMARKS

We are investigating on the some more Cycle related graphs which can be also labelled and proved similarly as Harmonic mean labelling and to find the generalised form.

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