N. Mohana, IJSRR 2019, 8(1), 2827-2841

Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

α −Cut of Trapezoidal Fuzzy Number Matrices

N. Mohana

Assistant Professor of Mathematics, A. V. C. College (Autonomous) Mannampandal, Mayiladuthurai, India.

ABSTRACT :

Introduced new elementary operators on α −cuts of Trapezoidal Fuzzy Numbers (TrFNs) are defined. We also defined some new operators on α −cuts of Trapezoidal Fuzzy Number matrices and examples (TrFMs). Using these operators, some important properties are proved.

KEYWORDS: Fuzzy Arithmetic, Fuzzy number, Trapezoidal fuzzy number (TrFN), Trapezoidal fuzzy matrix (TrFM), α −Cuts of Trapezoidal fuzzy matrix(TrFM).

***Corresponding author:**

Dr N. Mohana

Assistant Professor of Mathematics,

A,V.C.College (Autonomous)

Mannampandal, Mayiladuthurai, India.

I. INTRODUCTION

Real world decision making problems are often uncertain (or) vague in a number of ways. In 1965,Zadeh introduced the concept of fuzzy set theory to meet those problems. Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. The fuzziness can be represented by different ways. One of the most useful representation is the membership function depending on the nature of the membership function the fuzzy numbers can be classified in different forms, sucha as Triangular Fuzzy Numbers, Trapezoidal Fuzzy Numbers (TrFNs), Interval Fuzzy Numbers etc. Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced by M.G.Thomason[10] . Two new operators and some properties of fuzzy matrices over these new operators are given in Shyamal. A. K. and Pal. M [7].Two new operations and some applications of fuzzy matrices are given in Shyamal. A. K. and Pal. M [7, 8, 9].

Trapezoidal fuzzy number's (TrFNs) are frequently used in application. It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. To the best of our knowledge, no work is available on TrFMs, through a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below. Some properties of Constant of trapezoidal fuzzy number matrices by N.Mohana and R.Mani[4].Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory Overhinniko S.V [6] . Kim[1]presented some important results on determinant of square fuzzy matrices. He defined the determinant of a square fuzzy matrix and contributed with very research works Kim[2, 3]. Fuzzy matrices were introduced for the first time by Thomason[10] who discussed the convergence of power of fuzzy matrix. A note on Adjoint of trapezoidal fuzzy number matrices by N.Mohana and R.Mani[5] .

Firstly in section 2, we recall the definition of Trapezoidal fuzzy number and some new operations on trapezoidal fuzzy numbers (TrFNs). In section 3, we have reviewed the definition of trapezoidal fuzzy matrix (TrFM) and some new operations on Trapezoidal fuzzy matrices (TrFMs) and examples. In section 4, we defined the $\alpha -$cuts of Trapezoidal fuzzy matrix (TrFM) and Using these, some important properties proved. In section 5, conclusion is included.

II. PRELIMINARIES

In this section, We recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1 fuzzy set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x))\ ;\ x\in X\}$$

Here $\mu_A : X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2 Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x∈X such that $\mu_A(x)=1$.

Definition 2.3 Upper α −cut

For all $X, \alpha \in [0,1]$ the upper α −cut of X is denoted as $X^{(\alpha)}$ and is defined as

$$X^{(\alpha)} = \begin{cases} 1, & if X \geq \alpha \\ 0, & if X < \alpha \end{cases}.$$

Definition 2.4 Lower α −cut

For all $X, \alpha \in [0,1]$ the lower α −cut of X is denoted as $X_{(\alpha)}$ and is defined as

$$X_{(\alpha)} = \begin{cases} X, & if X \geq \alpha \\ 0, & if X < \alpha \end{cases}.$$

Definition 2.5 Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

(i) $\tilde{A}$ is normal (ii) $\tilde{A}$ is convex

(iii) The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.6 Trapezoidal fuzzy number

A fuzzy number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x) = \begin{cases} 0 & ;\ x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1} & ;\ a_1 \leq x \leq a_2 \\ 1 & ;\ a_2 \leq x \leq a_3 \end{cases}$$

$$\frac{a_4 - x}{a_4 - a_3} \quad ; \quad a_3 \le x \le a_4$$

$$0 \quad ; \quad x > a_4$$

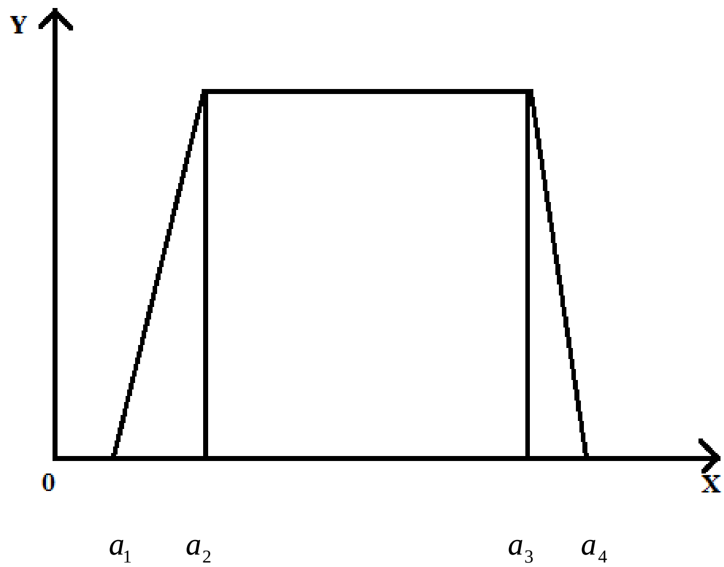

Fig 1: Trapezoidal Fuzzy Number

Example: 2.6.1

$\tilde{A}^{TzL} = (2,4,6,8)$ is a trapezoidal fuzzy number.

Definition 2.7 Arithmetic operations on trapezoidal fuzzy numbers (TrFNs)

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers (TrFNs) then we defined,

(1) $\tilde{A}^{TzL} \oplus \tilde{B}^{TzL} = (a_1 + b_1 - a_1 b_1, a_2 + b_2 - a_2 b_2, a_3 + b_3 - a_3 b_3, a_4 + b_4 - a_4 b_4)$

(2) $\tilde{A}^{TzL} \vee \tilde{B}^{TzL} = (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3, a_4 \vee b_4)$ where $X \vee Y$ means $\max\{x, y\}$.

i.e.,$X \vee Y = \max\{x, y\}$.

(3) $\tilde{A}^{TzL} \ominus \tilde{B}^{TzL} = (a_1 \ominus b_1, a_2 \ominus b_2, a_3 \ominus b_3, a_4 \ominus b_4)$ where $X \ominus Y = \begin{cases} x, & if\ x > y \\ 0, & if\ x \le y \end{cases}$.

(4) $\tilde{A}^{TzL} \ge \tilde{B}^{TzL}$ if $a_1 \ge b_1, a_2 \ge b_2, a_3 \ge b_3$, and $a_4 \ge b_4$.

(5) For all $\alpha \in [0,1]$,

$\tilde{A}^{TzL} \ge \alpha$ if $a_1 \ge \alpha, a_2 \ge \alpha, a_3 \ge \alpha, a_4 \ge \alpha$ and $\alpha \ge \tilde{A}^{TzL}$ if $\alpha \ge a_1, \alpha \ge a_2, \alpha \ge a_3, \alpha \ge a_4$.

Definition 2.8 Upper α −Cut trapezoidal fuzzy number

For $\alpha \in [0,1]$, the upper α −cut of Trapezoidal Fuzzy Number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is defined as $\tilde{A}^{TzL(\alpha)} = \left(a_1^{(\alpha)}, a_2^{(\alpha)}, a_3^{(\alpha)}, a_4^{(\alpha)}\right)$.

Definition 2.9 Lower α −Cut trapezoidal fuzzy number

For $\alpha \in [0,1]$, the lower α −cut of Trapezoidal Fuzzy Number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is defined as $\tilde{A}^{TzL}_{(\alpha)} = (a_{1(\alpha)}, a_{2(\alpha)}, a_{3(\alpha)}, a_{4(\alpha)})$.

Example: 2.9.1

Consider the trapezoidal fuzzy number $\tilde{A}^{TzL}$ as follows:

$\tilde{A}^{TzL} = (0.6,0.4,0.5,0.2)$. By taking $\alpha = 0.6$, we get

$\tilde{A}^{TzL}_{(\alpha)} = (0.6,0,0,0)$ and $\tilde{A}^{TzL(\alpha)} = (1,0,0,0)$.

III. TRAPEZOIDAL FUZZY MATRICES (TrFMs)

In this section, we introduced the trapezoidal fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition 3.1 Trapezoidal fuzzy matrix (TrFM)

A trapezoidal fuzzy matrix of order m×n is defined as A= $(\tilde{a}_{ij}^{TzL})_{m\times n}$, where a_{ij}= $(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is the ij^{th} element of A.

Definition 3.2 Operations on Trapezoidal Fuzzy Matrices (TrFMs)

As for classical matrices. We define the following operations on trapezoidal fuzzy matrices. Let A = ($\tilde{a}_{ij}^{TzL}$) and B =($\tilde{b}_{ij}^{TzL}$) be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

(1) $A \oplus B = (\tilde{a}_{ij}^{TzL} \oplus \tilde{b}_{ij}^{TzL})$.

(2) $A \vee B = (\tilde{a}_{ij}^{TzL} \vee \tilde{b}_{ij}^{TzL})$.

(3) $A \ominus B = (\tilde{a}_{ij}^{TzL} \ominus \tilde{b}_{ij}^{TzL})$.

(4) $A \geq B$ iff $\tilde{a}_{ij}^{TzL} \geq \tilde{b}_{ij}^{TzL}$ $\forall\, i = 1,2, \dots, m, j = 1,2, \dots, n$.

Example 3.2.1:

Let $A = \begin{bmatrix} (1,2,3,4) & (4,5,6,7) \\ (6,7,8,9) & (9,10,11,12) \end{bmatrix}$ and

$B = \begin{bmatrix} (2,4,6,8) & (4,6,8,10) \\ (6,7,9,11) & (3,5,7,9) \end{bmatrix}$ be two TrFMs.

Sol:

(1) $A \oplus B = \begin{bmatrix} (1,2,3,4) & (4,5,6,7) \\ (6,7,8,9) & (9,10,11,12) \end{bmatrix} \oplus \begin{bmatrix} (2,4,6,8) & (4,6,8,10) \\ (6,7,9,11) & (3,5,7,9) \end{bmatrix}$

$= \begin{bmatrix} (1,-2,-9,-20) & (-8,-19,-34,-53) \\ (-24,-35,-55,-79) & (-15,-35,-59,-87) \end{bmatrix}$.

(2) $A \ominus B = \begin{bmatrix} (1,2,3,4) & (4,5,6,7) \\ (6,7,8,9) & (9,10,11,12) \end{bmatrix} \ominus \begin{bmatrix} (2,4,6,8) & (4,6,8,10) \\ (6,7,9,11) & (3,5,7,9) \end{bmatrix}$

$$= \begin{bmatrix} (-1,-2,-3,-4) & (0,-1,-2,-3) \\ (0,0,-1,-2) & (6,5,4,3) \end{bmatrix}.$$

(3) $A \vee B = \begin{bmatrix} (1,2,3,4) & (4,5,6,7) \\ (6,7,8,9) & (9,10,11,12) \end{bmatrix} \vee \begin{bmatrix} (2,4,6,8) & (4,6,8,10) \\ (6,7,9,11) & (3,5,7,9) \end{bmatrix}$

$$= \begin{bmatrix} (2,4,6,8) & (4,6,8,10) \\ (6,7,9,11) & (9,10,11,12) \end{bmatrix}.$$

IV. α −CUTS OF TRAPEZOIDAL FUZZY MATRIX (TrFM)

In this section, we introduce the α −cuts in the fuzzy nature.

Definition 4.1 Upper α −Cut of trapezoidal fuzzy number matrix

The upper α −cut of a Trapezoidal Fuzzy Number Matrix $A = \left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$ is defined as $A^{(\alpha)} = \left(\tilde{a}_{ij}^{TzL(\alpha)}\right)_{m\times n}$.

Definition 4.2 Lower α −Cut of trapezoidal fuzzy number matrix

The lower α −cut of a Trapezoidal Fuzzy Number Matrix $A = \left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$ is defined as $A_{(\alpha)} = \left(\tilde{a}_{ij_{(\alpha)}}^{TzL}\right)_{m\times n}$.

Example: 4.2.1

Consider the trapezoidal fuzzy number matrix A as follows:

$$A = \begin{bmatrix} (0.5,0.1,0.4,0.3) & (0.1,0.7,0.2,0.6) \\ (0.1,0.3,0.5,0.2) & (0.2,0.6,0.3,0.65) \end{bmatrix}$$

Then by taking $\alpha = 0.5$, we get

$$A_{(\alpha)} = \begin{bmatrix} (0.5,0,0,0) & (0,0.7,0,0.6) \\ (0,0,0.5,0) & (0,0.6,0,0.65) \end{bmatrix}$$

$$A^{(\alpha)} = \begin{bmatrix} (1,0,0,0) & (0,1,0,1) \\ (0,0,1,0) & (0,1,0,1) \end{bmatrix}.$$

Theorem 4.1

For any two trapezoidal fuzzy number matrices A and B

(i) $(\boldsymbol{A} \ominus \boldsymbol{B})^{(\alpha)} \geq \boldsymbol{A}^{(\alpha)} \ominus \boldsymbol{B}^{(\alpha)}$

(ii) $(\boldsymbol{A} \vee \boldsymbol{B})^{(\alpha)} = \boldsymbol{A}^{(\alpha)} \vee \boldsymbol{B}^{(\alpha)}$

(iii) $(\boldsymbol{A} \oplus \boldsymbol{B})^{(\alpha)} \geq \boldsymbol{A}^{(\alpha)} \oplus \boldsymbol{B}^{(\alpha)}$

Proof

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$ where $\tilde{a}_{ij}^{TzL} = \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)$ and

$B = \left(\tilde{b}_{ij}^{TzL}\right)_{m\times n}$ where $\tilde{b}_{ij}^{TzL} = \left(b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}\right)$.

(i) Let $\tilde{R}_{ij}^{TzL}$ and $\tilde{M}_{ij}^{TzL}$ be the ij^{th} elements of $A^{(\alpha)} \ominus B^{(\alpha)}$ and $(A \ominus B)^{(\alpha)}$. $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \ominus \tilde{B}_{ij}^{TzL(\alpha)}$ and $\tilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$.

Case 1: $\widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \boldsymbol{\alpha}$

$$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$$

$$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha.$$

Therefore, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (1,1,1,1)$$

and

$$\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \ominus \tilde{B}_{ij}^{TzL(\alpha)}$$

$$= \left(a_{1ij}^{(\alpha)} \ominus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \ominus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \ominus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \ominus b_{4ij}^{(\alpha)}\right)$$

$$= (1 \ominus 1, 1 \ominus 1, 1 \ominus 1, 1 \ominus 1) = (0, 0, 0, 0).$$

i.e., $\widetilde{M}_{ij}^{TzL} > \tilde{R}_{ij}^{TzL}$.

Case 2: $\widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \boldsymbol{\alpha} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}}$

$$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$$

$$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}.$$

Then, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (1,1,1,1)$$

and $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \ominus \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \ominus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \ominus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \ominus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \ominus b_{4ij}^{(\alpha)}\right)$$

$$= (1 \ominus 0, 1 \ominus 0, 1 \ominus 0, 1 \ominus 0) = (1, 1, 1, 1).$$

i.e., $\widetilde{M}_{ij}^{TzL} = \tilde{R}_{ij}^{TzL}$.

Case 3: $\boldsymbol{\alpha} \geq \widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}}$

$$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$$

$$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}.$$

Here, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (0, 0, 0, 0)$$

And $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \ominus \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \ominus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \ominus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \ominus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \ominus b_{4ij}^{(\alpha)}\right)$$

$$= (0 \ominus 0, 0 \ominus 0, 0 \ominus 0, 0 \ominus 0) = (0, 0, 0, 0).$$

i.e., $\widetilde{M}_{ij}^{TzL} = \tilde{R}_{ij}^{TzL}$

∴In all the case, $\widetilde{M}_{ij}^{TzL} \geq \tilde{R}_{ij}^{TzL}$

$\therefore \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)} = \tilde{A}_{ij}^{TzL(\alpha)} \ominus \tilde{B}_{ij}^{TzL(\alpha)}$

$\therefore (A \ominus B)^{(\alpha)} \geq A^{(\alpha)} \ominus B^{(\alpha)}$.

(ii) Let $\tilde{C}_{ij}^{TzL}$ and $\widetilde{D}_{ij}^{TzL}$ be the ij^{th} elements of $A^{(\alpha)} \vee B^{(\alpha)}$ and $(A \vee B)^{(\alpha)}$.

$\therefore \tilde{C}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \vee \tilde{B}_{ij}^{TzL(\alpha)}$ and $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$.

Case 1: $\widetilde{A}_{ij}^{TzL} \geq \widetilde{B}_{ij}^{TzL} \geq \alpha$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$

$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha.$

Therefore, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (1,1,1,1)$$

And $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \vee \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \vee b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \vee b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \vee b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \vee b_{4ij}^{(\alpha)}\right)$$

$$= (1 \vee 1, 1 \vee 1, 1 \vee 1, 1 \vee 1) = (1,1,1,1).$$

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

Case 2: $\widetilde{A}_{ij}^{TzL} \geq \alpha \geq \widetilde{B}_{ij}^{TzL}$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}.$

Then, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (1,1,1,1)$$

And $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \vee \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \vee b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \vee b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \vee b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \vee b_{4ij}^{(\alpha)}\right)$$

$$= (1 \vee 0, 1 \vee 0, 1 \vee 0, 1 \vee 0) = (1,1,1,1).$$

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

Case 3: $\alpha \geq \widetilde{A}_{ij}^{TzL} \geq \widetilde{B}_{ij}^{TzL}$

$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}.$

Here, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} = \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (0,0,0,0)$$

And $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \vee \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \vee b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \vee b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \vee b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \vee b_{4ij}^{(\alpha)}\right)$$

$$= (0 \vee 0, 0 \vee 0, 0 \vee 0, 0 \vee 0) = (0,0,0,0).$$

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

∴In all the case, $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$

$\therefore \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)^{(\alpha)} = \tilde{A}_{ij}^{TzL(\alpha)} \vee \tilde{B}_{ij}^{TzL(\alpha)}$

$\therefore (A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$.

(iii) Let $\tilde{S}_{ij}^{TzL}$ and $\tilde{T}_{ij}^{TzL}$ be the ij^{th} elements of $A^{(\alpha)} \oplus B^{(\alpha)}$ and $(A \oplus B)^{(\alpha)}$

$\therefore \tilde{S}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \oplus \tilde{B}_{ij}^{TzL(\alpha)}$ and $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$.

Case 1: $\widetilde{A}_{ij}^{TzL} \geq \widetilde{B}_{ij}^{TzL} \geq \alpha$

$\therefore a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$

$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha.$

Therefore, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}^{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}^{(\alpha)}$$

$$\geq \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)^{(\alpha)} > \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) = (1,1,1,1)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \oplus \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \oplus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \oplus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \oplus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \oplus b_{4ij}^{(\alpha)}\right)$$

$$= \begin{pmatrix} a_{1ij}^{(\alpha)} + b_{1ij}^{(\alpha)} - a_{1ij}^{(\alpha)}.b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} + b_{2ij}^{(\alpha)} - a_{2ij}^{(\alpha)}.b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} + b_{3ij}^{(\alpha)} - a_{3ij}^{(\alpha)}.b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} + \\ b_{4ij}^{(\alpha)} - a_{4ij}^{(\alpha)}.b_{4ij}^{(\alpha)}) \end{pmatrix}$$

$$= (1 + 1 - 1, 1 + 1 - 1, 1 + 1 - 1, 1 + 1 - 1) = (1,1,1,1).$$

i.e., $\tilde{T}_{ij}^{TzL} > \tilde{S}_{ij}^{TzL}$.

Case 2: $\widetilde{A}_{ij}^{TzL} \geq \alpha \geq \widetilde{B}_{ij}^{TzL}$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}.$

Then, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}^{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}^{(\alpha)}$$

$$\geq (a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij})^{(\alpha)} \quad > \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) \qquad = (1,1,1,1)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \oplus \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \oplus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \oplus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \oplus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \oplus b_{4ij}^{(\alpha)}\right)$$

$$= \begin{pmatrix} a_{1ij}^{(\alpha)} + b_{1ij}^{(\alpha)} - a_{1ij}^{(\alpha)}.b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} + b_{2ij}^{(\alpha)} - a_{2ij}^{(\alpha)}.b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} + b_{3ij}^{(\alpha)} - a_{3ij}^{(\alpha)}.b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} + \\ b_{4ij}^{(\alpha)} - a_{4ij}^{(\alpha)}.b_{4ij}^{(\alpha)} \end{pmatrix}$$

$$= (1 + 0 - 0, 1 + 0 - 0, 1 + 0 - 0, 1 + 0 - 0) \quad = (1, 1, 1, 1).$$

i.e., $\tilde{T}_{ij}^{TzL} > \tilde{S}_{ij}^{TzL}$.

Case 3: $\boldsymbol{\alpha \geq \tilde{A}_{ij}^{TzL} \geq \tilde{B}_{ij}^{TzL}}$

$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}$.

Here, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}^{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}^{(\alpha)}$$

$$\geq (a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij})^{(\alpha)} \quad > \left(a_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)}\right) \qquad = (0, 0, 0, 0)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij}^{TzL(\alpha)} \oplus \tilde{B}_{ij}^{TzL(\alpha)}$

$$= \left(a_{1ij}^{(\alpha)} \oplus b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} \oplus b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} \oplus b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} \oplus b_{4ij}^{(\alpha)}\right)$$

$$= \begin{pmatrix} a_{1ij}^{(\alpha)} + b_{1ij}^{(\alpha)} - a_{1ij}^{(\alpha)}.b_{1ij}^{(\alpha)}, a_{2ij}^{(\alpha)} + b_{2ij}^{(\alpha)} - a_{2ij}^{(\alpha)}.b_{2ij}^{(\alpha)}, a_{3ij}^{(\alpha)} + b_{3ij}^{(\alpha)} - a_{3ij}^{(\alpha)}.b_{3ij}^{(\alpha)}, a_{4ij}^{(\alpha)} + \\ b_{4ij}^{(\alpha)} - a_{4ij}^{(\alpha)}.b_{4ij}^{(\alpha)} \end{pmatrix}$$

$$= (0 + 0 - 0, 0 + 0 - 0, 0 + 0 - 0, 0 + 0 - 0) = (0, 0, 0, 0).$$

$\therefore \tilde{T}_{ij}^{TzL} = \tilde{S}_{ij}^{TzL}$.

$\therefore$In all the case, $\tilde{T}_{ij}^{TzL} \geq \tilde{S}_{ij}^{TzL}$

$\therefore \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)^{(\alpha)} \geq \tilde{A}_{ij}^{TzL(\alpha)} \oplus \tilde{B}_{ij}^{TzL(\alpha)}$

$\therefore (A \oplus B)^{(\alpha)} \geq A^{(\alpha)} \oplus B^{(\alpha)}$.

Theorem 4.2

For any two trapezoidal fuzzy number matrices A and B

(iv) $(\boldsymbol{A} \ominus \boldsymbol{B})_{(\alpha)} \geq \boldsymbol{A}_{(\alpha)} \ominus \boldsymbol{B}_{(\alpha)}$

(v) $(\boldsymbol{A} \vee \boldsymbol{B})_{(\alpha)} = \boldsymbol{A}_{(\alpha)} \vee \boldsymbol{B}_{(\alpha)}$

(vi) $(\boldsymbol{A} \oplus \boldsymbol{B})_{(\alpha)} \geq \boldsymbol{A}_{(\alpha)} \oplus \boldsymbol{B}_{(\alpha)}$

Proof

Let$A = \left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$where $\tilde{a}_{ij}^{TzL} = \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)$ and

$B = \left(\tilde{b}_{ij}^{TzL}\right)_{m\times n}$where $\tilde{b}_{ij}^{TzL} = \left(b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}\right)$.

(iv) Let $\tilde{R}_{ij}^{TzL}$ and $\widetilde{M}_{ij}^{TzL}$ be the ij^{th} elements of $A_{(\alpha)} \ominus B_{(\alpha)}$ and $(A \ominus B)_{(\alpha)}$.

$\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij\,(\alpha)}^{TzL} \ominus \tilde{B}_{ij\,(\alpha)}^{TzL}$ and $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$.

Case 1: $\widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \boldsymbol{\alpha}$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$

$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha.$

Therefore, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (1,1,1,1)$

and $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij\,(\alpha)}^{TzL} \ominus \tilde{B}_{ij\,(\alpha)}^{TzL}$

$= \left(a_{1ij(\alpha)} \ominus b_{1ij(\alpha)}, a_{2ij(\alpha)} \ominus b_{2ij(\alpha)}, a_{3ij(\alpha)} \ominus b_{3ij(\alpha)}, a_{4ij(\alpha)} \ominus b_{4ij(\alpha)}\right)$

$= (1 \ominus 1, 1 \ominus 1, 1 \ominus 1, 1 \ominus 1) = (0,0,0,0).$

i.e., $\widetilde{M}_{ij}^{TzL} > \tilde{R}_{ij}^{TzL}$.

Case 2: $\widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \boldsymbol{\alpha} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}}$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}.$

Then, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (1,1,1,1)$

and $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij\,(\alpha)}^{TzL} \ominus \tilde{B}_{ij\,(\alpha)}^{TzL}$

$= \left(a_{1ij(\alpha)} \ominus b_{1ij(\alpha)}, a_{2ij(\alpha)} \ominus b_{2ij(\alpha)}, a_{3ij(\alpha)} \ominus b_{3ij(\alpha)}, a_{4ij(\alpha)} \ominus b_{4ij(\alpha)}\right)$

$= (1 \ominus 0, 1 \ominus 0, 1 \ominus 0, 1 \ominus 0) = (1,1,1,1).$

i.e., $\widetilde{M}_{ij}^{TzL} = \tilde{R}_{ij}^{TzL}$.

Case 3: $\boldsymbol{\alpha} \geq \widetilde{\boldsymbol{A}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}} \geq \widetilde{\boldsymbol{B}}_{\boldsymbol{ij}}^{\boldsymbol{TzL}}$

$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}$.

Here, $\widetilde{M}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) =$ (0, 0, 0, 0)

and $\tilde{R}_{ij}^{TzL} = \tilde{A}_{ij\,(\alpha)}^{TzL} \ominus \tilde{B}_{ij\,(\alpha)}^{TzL}$

$= \left(a_{1ij(\alpha)} \ominus b_{1ij(\alpha)}, a_{2ij(\alpha)} \ominus b_{2ij(\alpha)}, a_{3ij(\alpha)} \ominus b_{3ij(\alpha)}, a_{4ij(\alpha)} \ominus b_{4ij(\alpha)}\right)$

$= (0 \ominus 0, 0 \ominus 0, 0 \ominus 0, 0 \ominus 0) = (0, 0, 0, 0)$.

i.e., $\widetilde{M}_{ij}^{TzL} = \tilde{R}_{ij}^{TzL}$

$\therefore$In all the case, $\widetilde{M}_{ij}^{TzL} \geq \tilde{R}_{ij}^{TzL}$

$\therefore \left(\tilde{A}_{ij}^{TzL} \ominus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)} = \tilde{A}_{ij\,(\alpha)}^{TzL} \ominus \tilde{B}_{ij\,(\alpha)}^{TzL}$

$\therefore (A \ominus B)_{(\alpha)} \geq A_{(\alpha)} \ominus B_{(\alpha)}$.

(v) Let $\tilde{C}_{ij}^{TzL}$ and $\widetilde{D}_{ij}^{TzL}$ be the ij^{th} elements of $A_{(\alpha)} \vee B_{(\alpha)}$ and $(A \vee B)_{(\alpha)}$.

$\therefore \tilde{C}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \vee \tilde{B}_{ij(\alpha)}^{TzL}$ and $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$.

Case 1: $\tilde{A}_{ij}^{TzL} \geq \tilde{B}_{ij}^{TzL} \geq \alpha$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$

$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha$.

Therefore, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (1,1,1,1)$

and $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \vee \tilde{B}_{ij(\alpha)}^{TzL}$

$= \left(a_{1ij(\alpha)} \vee b_{1ij(\alpha)}, a_{2ij(\alpha)} \vee b_{2ij(\alpha)}, a_{3ij(\alpha)} \vee b_{3ij(\alpha)}, a_{4ij(\alpha)} \vee b_{4ij(\alpha)}\right)$

$= (1 \vee 1, 1 \vee 1, 1 \vee 1, 1 \vee 1) = (1, 1, 1, 1)$.

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

Case 2: $\tilde{A}_{ij}^{TzL} \geq \alpha \geq \tilde{B}_{ij}^{TzL}$

$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$

$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}$.

Then, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (1,1,1,1)$

and $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \vee \tilde{B}_{ij(\alpha)}^{TzL}$

$$= \left(a_{1ij(\alpha)} \vee b_{1ij(\alpha)}, a_{2ij(\alpha)} \vee b_{2ij(\alpha)}, a_{3ij(\alpha)} \vee b_{3ij(\alpha)}, a_{4ij(\alpha)} \vee b_{4ij(\alpha)}\right)$$

$$= (1 \vee 0, 1 \vee 0, 1 \vee 0, 1 \vee 0) = (1, 1, 1, 1).$$

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

Case 3: $\boldsymbol{\alpha \geq \widetilde{A}_{ij}^{TzL} \geq \widetilde{B}_{ij}^{TzL}}$

$$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$$

$$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}.$$

Here, $\widetilde{D}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$$= \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} = \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (0, 0, 0, 0)$$

and $\tilde{C}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \vee \tilde{B}_{ij(\alpha)}^{TzL}$

$$= \left(a_{1ij(\alpha)} \vee b_{1ij(\alpha)}, a_{2ij(\alpha)} \vee b_{2ij(\alpha)}, a_{3ij(\alpha)} \vee b_{3ij(\alpha)}, a_{4ij(\alpha)} \vee b_{4ij(\alpha)}\right)$$

$$= (0 \vee 0, 0 \vee 0, 0 \vee 0, 0 \vee 0) = (0, 0, 0, 0).$$

i.e., $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$.

∴In all the case, $\widetilde{D}_{ij}^{TzL} = \tilde{C}_{ij}^{TzL}$

$$\therefore \left(\tilde{A}_{ij}^{TzL} \vee \tilde{B}_{ij}^{TzL}\right)_{(\alpha)} = \tilde{A}_{ij(\alpha)}^{TzL} \vee \tilde{B}_{ij(\alpha)}^{TzL}$$

$$\therefore (A \vee B)_{(\alpha)} = A_{(\alpha)} \vee B_{(\alpha)}.$$

(vi) Let $\tilde{S}_{ij}^{TzL}$ and $\tilde{T}_{ij}^{TzL}$ be the ij^{th} elements of $A_{(\alpha)} \oplus B_{(\alpha)}$ and $(A \oplus B)_{(\alpha)}$.

$\therefore \tilde{S}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \oplus \tilde{B}_{ij(\alpha)}^{TzL}$ and $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$.

Case 1: $\boldsymbol{\widetilde{A}_{ij}^{TzL} \geq \widetilde{B}_{ij}^{TzL} \geq \alpha}$

$$\therefore a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij} \geq \alpha$$

$$\therefore a_{1ij} \geq b_{1ij} \geq \alpha, a_{2ij} \geq b_{2ij} \geq \alpha, a_{3ij} \geq b_{3ij} \geq \alpha, a_{4ij} \geq b_{4ij} \geq \alpha.$$

Therefore, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}_{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}_{(\alpha)}$$

$$\geq \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}\right)_{(\alpha)} > \left(a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}\right) = (1,1,1,1)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \oplus \tilde{B}_{ij(\alpha)}^{TzL}$

$$= \left(a_{1ij(\alpha)} \oplus b_{1ij(\alpha)}, a_{2ij(\alpha)} \oplus b_{2ij(\alpha)}, a_{3ij(\alpha)} \oplus b_{3ij(\alpha)}, a_{4ij(\alpha)} \oplus b_{4ij(\alpha)}\right)$$

$$= \begin{pmatrix} a_{1ij(\alpha)} + b_{1ij(\alpha)} - a_{1ij(\alpha)}.b_{1ij(\alpha)}, a_{2ij(\alpha)} + b_{2ij(\alpha)} - a_{2ij(\alpha)}.b_{2ij(\alpha)}, \\ a_{3ij(\alpha)} + b_{3ij(\alpha)} - a_{3ij(\alpha)}.b_{3ij(\alpha)}, \\ a_{4ij(\alpha)} + b_{4ij(\alpha)} - a_{4ij(\alpha)}.b_{4ij(\alpha)} \end{pmatrix}$$

$$= (1 + 1 - 1, 1 + 1 - 1, 1 + 1 - 1, 1 + 1 - 1) = (1, 1, 1, 1).$$

i.e., $\tilde{T}_{ij}^{TzL} > \tilde{S}_{ij}^{TzL}$.

Case 2: $\tilde{A}_{ij}^{TzL} \geq \alpha \geq \tilde{B}_{ij}^{TzL}$

$$a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq \alpha \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$$

$$\therefore a_{1ij} \geq \alpha \geq b_{1ij}, a_{2ij} \geq \alpha \geq b_{2ij}, a_{3ij} \geq \alpha \geq b_{3ij}, a_{4ij} \geq \alpha \geq b_{4ij}.$$

Then, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}_{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}_{(\alpha)}$$

$$\geq (a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij})_{(\alpha)} > (a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}) = (1,1,1,1)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \oplus \tilde{B}_{ij(\alpha)}^{TzL}$

$$= (a_{1ij(\alpha)} \oplus b_{1ij(\alpha)}, a_{2ij(\alpha)} \oplus b_{2ij(\alpha)}, a_{3ij(\alpha)} \oplus b_{3ij(\alpha)}, a_{4ij(\alpha)} \oplus b_{4ij(\alpha)})$$

$$= \begin{pmatrix} a_{1ij(\alpha)} + b_{1ij(\alpha)} - a_{1ij(\alpha)}.b_{1ij(\alpha)}, a_{2ij(\alpha)} + b_{2ij(\alpha)} - a_{2ij(\alpha)}.b_{2ij(\alpha)}, \\ a_{3ij(\alpha)} + b_{3ij(\alpha)} - a_{3ij(\alpha)}.b_{3ij(\alpha)}, \\ a_{4ij(\alpha)} + b_{4ij(\alpha)} - a_{4ij(\alpha)}.b_{4ij(\alpha)} \end{pmatrix}$$

$$= (1 + 0 - 0, 1 + 0 - 0, 1 + 0 - 0, 1 + 0 - 0) = (1, 1, 1, 1).$$

i.e., $\tilde{T}_{ij}^{TzL} > \tilde{S}_{ij}^{TzL}$.

Case 3: $\alpha \geq \tilde{A}_{ij}^{TzL} \geq \tilde{B}_{ij}^{TzL}$

$$\alpha \geq a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij} \geq b_{1ij}, b_{2ij}, b_{3ij}, b_{4ij}$$

$$\therefore \alpha \geq a_{1ij} \geq b_{1ij}, \alpha \geq a_{2ij} \geq b_{2ij}, \alpha \geq a_{3ij} \geq b_{3ij}, \alpha \geq a_{4ij} \geq b_{4ij}.$$

Here, $\tilde{T}_{ij}^{TzL} = \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)}$

$$= \begin{pmatrix} a_{1ij} + b_{1ij} - a_{1ij}.b_{1ij}, a_{2ij} + b_{2ij} - a_{2ij}.b_{2ij}, a_{3ij} + b_{3ij} - a_{3ij}.b_{3ij}, a_{4ij} + \\ b_{4ij} - a_{4ij}.b_{4ij} \end{pmatrix}_{(\alpha)}$$

$$= \begin{pmatrix} a_{1ij} + b_{1ij}(1 - a_{1ij}), a_{2ij} + b_{2ij}(1 - a_{2ij}), a_{3ij} + b_{3ij}(1 - a_{3ij}), a_{4ij} + \\ b_{4ij}(1 - a_{4ij}) \end{pmatrix}_{(\alpha)}$$

$$\geq (a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij})_{(\alpha)} > (a_{1ij\,(\alpha)}, a_{2ij\,(\alpha)}, a_{3ij\,(\alpha)}, a_{4ij\,(\alpha)}) = (0, 0, 0, 0)$$

and $\tilde{S}_{ij}^{TzL} = \tilde{A}_{ij(\alpha)}^{TzL} \oplus \tilde{B}_{ij(\alpha)}^{TzL}$

$$= (a_{1ij(\alpha)} \oplus b_{1ij(\alpha)}, a_{2ij(\alpha)} \oplus b_{2ij(\alpha)}, a_{3ij(\alpha)} \oplus b_{3ij(\alpha)}, a_{4ij(\alpha)} \oplus b_{4ij(\alpha)})$$

$$= \begin{pmatrix} a_{1ij(\alpha)} + b_{1ij(\alpha)} - a_{1ij(\alpha)}.b_{1ij(\alpha)}, a_{2ij(\alpha)} + b_{2ij(\alpha)} - a_{2ij(\alpha)}.b_{2ij(\alpha)}, \\ a_{3ij(\alpha)} + b_{3ij(\alpha)} - a_{3ij(\alpha)}.b_{3ij(\alpha)}, \\ a_{4ij(\alpha)} + b_{4ij(\alpha)} - a_{4ij(\alpha)}.b_{4ij(\alpha)} \end{pmatrix}$$

$$= (0 + 0 - 0, 0 + 0 - 0, 0 + 0 - 0, 0 + 0 - 0) = (0, 0, 0, 0).$$

$\therefore \tilde{T}_{ij}^{TzL} = \tilde{S}_{ij}^{TzL}$.

$\therefore$In all the case, $\tilde{T}_{ij}^{TzL} \geq \tilde{S}_{ij}^{TzL}$

$\therefore \left(\tilde{A}_{ij}^{TzL} \oplus \tilde{B}_{ij}^{TzL}\right)_{(\alpha)} \geq \tilde{A}_{ij(\alpha)}^{TzL} \oplus \tilde{B}_{ij(\alpha)}^{TzL}$

$\therefore (A \oplus B)_{(\alpha)} \geq A_{(\alpha)} \oplus B_{(\alpha)}$.

CONCLUSION

In this article, We have concentrate the α −Cuts of Trapezoidal Fuzzy Matrices are defined and also some important properties of α −Cuts of TrFMs are proved. Also the theories of the discussed Tr FMs may be utilized in further works.

REFERENCES

1. Kim. J. B., Determinant theory for fuzzy and Boolean matrices, Congresses numerantium, 1988;273-276.
2. Kim. J. B., A Baartmans Determinat Theory for Fuzzy Matrices, Fuzzy Sets and Systems,1989; 29: 349-356.
3. Kim. J. B., Inverse of Boolean Matrices, Bull. Inst. Math. Acod, Science, 1984; 12(2); 125-128.
4. Mohana. N. and Mani. R., Some Properties of Constant of Trapezoidal Fuzzy Matrices., Internation
5. Journal for Science and Advance Research In Technology, 2018; 4(2): 102-108.
6. Mohana. N. and Mani. R., A Note on Adjoint of Trapezoidal Fuzzy Number Matrices., Internation
7. Journal of Innovative Research in Technology., Feb. 2018; 4(9): 31-37.
8. Overhinniko S. V., Structure of fuzzy relations, Fuzzy Sets and Systems, 1981; 6: 169-195.
9. Shyamal. A.K. and Pal. M., Two new operators on fuzzy matrices, J.Applied Mathematics and computing , 2004; 13: 91-107.
10. Shyamal. A. K. and Pal. M., Distance between fuzzy matrices and its applications, Actasiencia Indica, 2005; XXXI-M(1): 199-204.
11. Shyamal. A. K. and Pal. M., Triangular fuzzy matrices Iranian journal of fuzzy systems, 2007; 4(1):75-87.
12. Thomson. M. G., Convergence of power of the fuzzy matrix, J.Math, Anal.Appl. 1997; 57: 476- 2480.