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Viscoelastic Displacements of Two Welded Half-Spaces Caused by a Dipole Source

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ABSTRACT

The correspondence principle of linear viscoelasticity is used to derive the displacement fields of an elastic half-space overlying a viscoelastic half-space caused by a dipole source, assuming the medium to be elastic in dilatation and Kelvin, Maxwell or SLS (Standard linear solid) type viscoelastic in distortion. The results are valid for arbitrary values of the relaxation time and a change in the rigidity of the two half-spaces. The variation of the viscoelastic displacements with the epicentral distance as well as with the different relaxation time are studied and shown graphically.

KEYWORDS: Viscoelastic, Dipole, Maxwell, Kelvin model.

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INTRODUCTION

The Papkovich-Neuber displacement potential functions for an arbitrary point force acting in an infinite medium consisting of two welded elastic half-spaces obtained by Rongved[1]. Heaton and Heaton[2] obtained the deformation field produced by point force and force couples embedded in two Poissonian half-space by using Papkovich-Neuber displacement potential functions. Rosenman and Singh[3] applied the correspondence principle of linear viscoelasticity to derive the quasi-static displacement field in a Maxwellian viscoelastic half-space. The correspondence principle of linear viscoelasticity has been extensively used by many authors (e.g.[4-8]) to calculate the quasi-static deformation of a viscoelastic half-space by a point or extended sources. The Galerkin vector approach has been used by Singh and Singh[9] to obtain the displacement field due to various seismic sources in a homogeneous, isotropic, perfectly elastic half-space and then the correspondence principle of linear viscoelasticity is used to obtain the quasi-static displacement, stains, and stresses. Singh and Singh[10] gave a simple procedure to obtain the quasi-static field in a viscoelastic half-space. The displacement and stress fields due to a point displacement dislocation located at an arbitrary point of a two- phase medium consisting of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact have been obtained by Kumari et al.[11]. Four axially symmetric sources, namely, a vertical force, a vertical dipole, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in an elastic half-space were considered by Singh et al.[12] to model the ground deformation in volcanic areas. The displacement and strain fields due to these four sources are compared with the corresponding fields due to a center of dilatation. The deformation fields due to five axially symmetric sources, namely, a vertical force, a vertical dipole, a center of dilatation, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in two welded elastic half-spaces obtained by Singh et al.[13].

Some materials usually have elastic as well as viscous property, therefore a theoretical study of viscoelasticity is an important subject in applied mechanics. A Viscoelastic medium has been used by many authors in various branches of science and technology particularly in geophysics and seismology.

In the present paper, we have obtained the displacement components due to a dipole in a homogeneous, isotropic, elastic half-space overlying homogeneous, isotropic, viscoelastic half-space.

THEORY

Consider a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space ($z > 0$, Medium-1) with the elastic constants $\lambda_1, \mu_1, \sigma_1$ in welded contact with other homogeneous, isotropic, viscoelastic half-space ($z < 0$, Medium-2) with the elastic constants $\lambda_2, \mu_2, \sigma_2$.

Let the point source of unit magnitude F_o is located at the point (0, 0, c) in Medium-1.

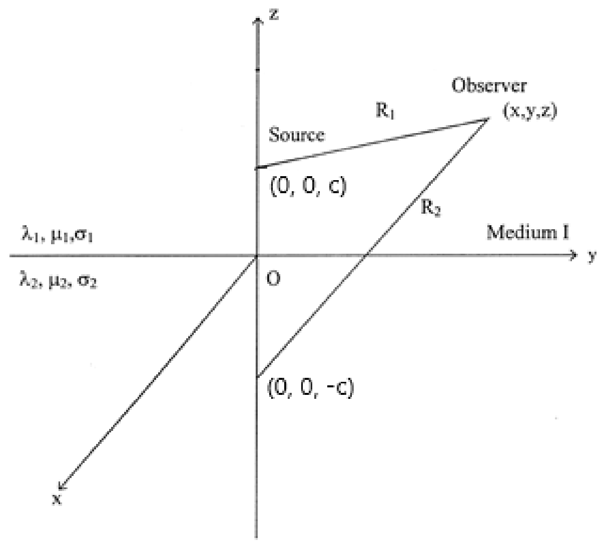


Fig. 1. Geometry of a point source in two welded half-spaces

The solution will be given in terms of the Papkovich- Neuber displacement potentials which derive the displacement through the relation

$$2\mu u_i = (\kappa + 1)\Phi_i - (x_k \Phi_k + \Psi)_{,i} \quad (1)$$

Where $x_i = (x, y, z)$ is the position vector, μ - shear modulus, $\kappa = 3 - 4\sigma$ where σ is the Poisson's Ratio.

In case of nobody forces, the potentials Φ_i *and* Ψ must be harmonic:

$$\nabla^2 \Phi_x = \nabla^2 \Phi_y = \nabla^2 \Phi_z = \nabla^2 \Psi = 0.$$

The Papkovich- Neuber displacement potentials for a dipole in the x, y, and z-directions have been used by Kumari et al.[11] in equation (1) to obtain the elastic solutions in an elastic half-space overlying elastic half-space.

The viscoelastic solution is obtained from the elastic solution (Kumari et al.[11]) by first applying the correspondence principle of linear viscoelasticity and then inverting the Laplace transformed solution. These solutions are to be obtained by considering homogenous, isotropic, elastic half-space overlying the homogenous, isotropic, viscoelastic half-space. For this, we consider $\kappa_2 = \kappa, \mu_2 = \mu$, and κ_1 *and* μ_1 as constants.

Following are the various combinations of elastic moduli μ *and* κ which occur in the expressions of elastic displacements:

$J_3 = \frac{1}{\mu + a}$, $J_4 = \frac{1}{\mu + \mu_1}$, $J_5 = \frac{1}{\mu + \kappa\, \mu_1}$, *where* $a = \frac{\mu_1}{\kappa_1}$ *and* $J_3, J_4,$ *and* J_5 are auxiliary functions.

We have derived these auxiliary functions by considering the material is elastic in dilatation and viscoelastic in distortion for the three viscoelastic models, namely, a Kelvin Model, a Maxwell Model, and a Standard Linear Solid Model. It is assumed that the time t>0 and the source time function is the Heaviside step function H (t).

DISPLACEMENT FIELD

Displacement components for the dipole (11) in the x-direction (horizontal dipole (HD))

Following are the displacement components for the horizontal dipole:

$$\hat{u}_{(1)x}^{HD}(t) = \frac{F_0 x}{4\pi(\kappa_1+1)\mu_1}\left\{\begin{aligned} &\frac{(\kappa_1-2)}{R_1^3} + \frac{3x^2}{R_1^5} + \frac{\left(2\mu_1\hat{J}_4(t)-1\right)(\kappa_1-2)}{R_2^3} + \frac{3\left[\kappa_1 x^2\left(2\mu_1\hat{J}_4(t)-1\right)+6cz\left((\kappa_1+1)a\hat{J}_3(t)-1\right)\right]}{\kappa_1 R_2^5} \\ &-\frac{3ox^2cz\left((\kappa_1+1)a\hat{J}_3(t)-1\right)}{\kappa_1 R_2^7} - \frac{\left((\kappa_1+1)a\hat{J}_3(t)-2\mu_1\hat{J}_4(t)\right)(z+c)}{R_2^2 R_4}\left(\frac{3}{R_2}+\frac{3}{R_4}-\frac{3x^2}{R_2^3}-\frac{3x^2}{R_2^2 R_4}-\frac{2x^2}{R_2 R_4^2}\right) \\ &+\frac{\left[\kappa_1(\kappa_1+1)a\hat{J}_3(t)-4\kappa_1\mu_1\hat{J}_4(t)+\kappa\mu_1(\kappa_1+1)\hat{J}_5(t)\right]}{2R_2R_4^2}\left(3-\frac{x^2}{R_2^2}-\frac{2x^2}{R_2R_4}\right) \end{aligned}\right\}$$

$$\hat{u}_{(1)y}^{HD}(t) = \frac{-F_0 y}{4\pi(\kappa_1+1)\mu_1}\left\{\begin{aligned} &\frac{1}{R_1^3} - \frac{3x^2}{R_1^5} + \frac{\left(2\mu_1\hat{J}_4(t)-1\right)}{R_2^3} - \frac{3\left[\kappa_1 x^2\left(2\mu_1\hat{J}_4(t)-1\right)+2cz\left((\kappa_1+1)a\hat{J}_3(t)-1\right)\right]}{\kappa_1 R_2^5} + \frac{3ox^2cz\left((\kappa_1+1)a\hat{J}_3(t)-1\right)}{\kappa_1 R_2^7} \\ &+\frac{\left((\kappa_1+1)a\hat{J}_3(t)-2\mu_1\hat{J}_4(t)\right)(z+c)}{R_2^2 R_4}\left(\frac{1}{R_2}+\frac{1}{R_4}-\frac{3x^2}{R_2^3}-\frac{3x^2}{R_2^2 R_4}-\frac{2x^2}{R_2 R_4^2}\right) \\ &-\frac{\left[\kappa_1(\kappa_1+1)a\hat{J}_3(t)-4\kappa_1\mu_1\hat{J}_4(t)+\kappa\mu_1(\kappa_1+1)\hat{J}_5(t)\right]}{2R_2R_4^2}\left(1-\frac{x^2}{R_2^2}-\frac{2x^2}{R_2R_4}\right) \end{aligned}\right\}$$

$$\hat{u}_{(1)z}^{HD}(t)=\frac{-F_0}{4\pi(\kappa_1+1)\mu_1}\left\{\begin{array}{l}(z-c)\left(\frac{1}{R_1^3}-\frac{3x^2}{R_1^5}\right)+\frac{(z-c)\left((\kappa_1+1)a\hat{J}_3(t)-1\right)}{R_2^3}-\frac{3\left((\kappa_1+1)a\hat{J}_3(t)-1\right)\left[\kappa_1x^2(z-c)+2cz(z+c)\right]}{\kappa_1R_2^5}\\+\frac{3\sigma x^2cz(z+c)\left((\kappa_1+1)a\hat{J}_3(t)-1\right)}{\kappa_1R_2^7}+\frac{(\kappa_1+1)\left(\kappa_1a\hat{J}_3(t)-\kappa\mu_1\hat{J}_5(t)\right)}{2R_2R_4}\left(1-\frac{x^2}{R_2^2}-\frac{x^2}{R_2R_4}\right)\end{array}\right\}$$

$$\hat{u}_{(2)x}^{HD}(t)=\frac{F_0x}{4\pi}\left\{\begin{array}{l}\frac{2(\kappa-1)\hat{J}_4(t)}{(\kappa+1)R_1^3}+\frac{6x^2\hat{J}_4(t)}{(\kappa+1)R_1^5}+\frac{\left[z\left(\hat{J}_5(t)-\frac{2\hat{J}_4(t)}{(\kappa+1)}\right)-c\left(\frac{\hat{J}_3(t)}{\kappa_1}-\frac{2\hat{J}_4(t)}{(\kappa+1)}\right)\right]}{R_1^2R_3}\left(\frac{3}{R_1}+\frac{3}{R_3}-\frac{3x^2}{R_1^3}-\frac{3x^2}{R_1^2R_3}-\frac{2x^2}{R_1R_3^2}\right)\\+\frac{\left[\hat{J}_3(t)-\frac{4\kappa\hat{J}_4(t)}{(\kappa+1)}+\kappa\hat{J}_5(t)\right]}{2R_1R_3^2}\left(3-\frac{x^2}{R_1^2}-\frac{2x^2}{R_1R_3}\right)\end{array}\right\}$$

$$\hat{u}_{(2)y}^{HD}(t)=\frac{-F_0y}{4\pi}\left\{\begin{array}{l}\frac{2\hat{J}_4(t)}{(\kappa+1)}\left[\frac{1}{R_1^3}-\frac{3x^2}{R_1^5}\right]-\frac{\left[z\left(\hat{J}_5(t)-\frac{2\hat{J}_4(t)}{(\kappa+1)}\right)-c\left(\frac{\hat{J}_3(t)}{\kappa_1}-\frac{2\hat{J}_4(t)}{(\kappa+1)}\right)\right]}{R_1^2R_3}\left(\frac{1}{R_1}+\frac{1}{R_3}-\frac{3x^2}{R_1^3}-\frac{3x^2}{R_1^2R_3}-\frac{2x^2}{R_1R_3^2}\right)\\-\frac{\left[\hat{J}_3(t)-\frac{4\kappa\hat{J}_4(t)}{(\kappa+1)}+\kappa\hat{J}_5(t)\right]}{2R_1R_3^2}\left(1-\frac{x^2}{R_1^2}-\frac{2x^2}{R_1R_3}\right)\end{array}\right\}$$

$$\hat{u}_{(2)z}^{HD}(t)=\frac{-F_0}{4\pi}\left\{\left[z\hat{J}_5(t)-c\frac{\hat{J}_3(t)}{\kappa_1}\right]\left(\frac{1}{R_1^3}-\frac{3x^2}{R_1^5}\right)+\frac{\left[\hat{J}_3(t)-\kappa\hat{J}_5(t)\right]}{2R_1R_3}\left(1-\frac{x^2}{R_1^2}-\frac{2x^2}{R_1R_3}\right)\right\}$$

Displacement components for the dipole (22) in the y-direction

The corresponding displacement components for the dipole in the y-direction (dipole (22)) at point (0, 0, c) of an elastic half-space in welded contact with another viscoelastic half-space can be obtained from the corresponding expressions for dipole (11) on interchanging x and y.

Displacement components for the dipole (33) in the z-direction (vertical dipole (VD))

Following are the displacement components for the vertical dipole:

$$\hat{u}_{(1)x}^{VD}(t)=\frac{F_0x}{4\pi(\kappa_1+1)\mu_1}\left\{\begin{array}{l}\frac{-1}{R_1^3}+\frac{3(z-c)^2}{R_1^5}-\frac{\left[(2-\kappa_1)(\kappa_1+1)a\hat{J}_3(t)+\kappa\mu_1(\kappa_1+1)\hat{J}_5(t)-2\right]}{2R_2^3}-\frac{3\left((\kappa_1+1)a\hat{J}_3(t)-1\right)\left[\kappa_1\left(z^2-c^2\right)-2z(z+2c)\right]}{\kappa_1R_2^5}\\-\frac{3\sigma(z+c)^2cz\left((\kappa_1+1)a\hat{J}_3(t)-1\right)}{\kappa_1R_2^7}\end{array}\right\}$$

$$\hat{u}^{VD}_{(1)z}(t)=\frac{F_0}{4\pi(\kappa_1+1)\mu_1}\left\{\begin{array}{l}(z-c)\left[\frac{\kappa_1-2}{R_1^3}+\frac{3(z-c)^2}{R_1^5}\right]+\frac{\frac{4((\kappa_1+1)a\hat{J}_3(t)-1)}{\kappa_1}[\kappa_1(z+c)-z]-(z+c)[(\kappa_1+1)(a\kappa_1\hat{J}_3(t)+\kappa\mu_1\hat{J}_5(t))-2\kappa_1]}{2R_2^3}\\ -\frac{3((\kappa_1+1)a\hat{J}_3(t)-1)[\kappa_1(z+c)^2-2z(z+4c)](z+c)}{\kappa_1 R_2^5}-\frac{30(z+c)^3cz((\kappa_1+1)a\hat{J}_3(t)-1)}{\kappa_1 R_2^7}\end{array}\right\}$$

$$\hat{u}^{VD}_{(2)x}(t)=\frac{F_0x}{4\pi}\left\{\left[\frac{\frac{(\kappa_1-2)\hat{J}_3(t)}{\kappa_1}-\kappa\hat{J}_5(t)}{2R_1^3}\right]+\frac{3(z-c)\left[z\hat{J}_5(t)-\frac{c\hat{J}_3(t)}{\kappa_1}\right]}{R_1^5}\right\}$$

$$\hat{u}^{VD}_{(2)z}(t)=\frac{F_0}{4\pi}\left\{(z-c)\left[\frac{\frac{(\kappa_1-2)\hat{J}_3(t)}{\kappa_1}+\kappa\hat{J}_5(t)}{2R_1^3}\right]-\left[z\hat{J}_5(t)-\frac{c\hat{J}_3(t)}{\kappa_1}\right]\left(\frac{1}{R_1^3}-\frac{3(z-c)^2}{R_1^5}\right)\right\}$$

For $u_{(1)y}$ and $u_{(2)y}$ we interchange x and y in $u_{(1)x}$ and $u_{(2)x}$ respectively.

Where $u_{(1)x}, u_{(1)y}, u_{(1)z}$ are the displacements in medium-1, and $u_{(2)x}, u_{(2)y}, u_{(2)z}$ are the displacements for the dipole in medium-2.

In above relations $\kappa_1 = 3-4\sigma_1$, $\kappa_2 = \kappa = 3-4\sigma_2$, $m = \frac{\mu_2}{\mu_1} = \frac{\mu}{\mu_1}$

$$A = \frac{1-m}{1+m\kappa_1}, B = \frac{\kappa - m\kappa_1}{\kappa+m}, S = \frac{1-m}{1+m}, T = \frac{2m(1+\kappa_1)}{(1+m)(1+\kappa)}$$

$$R_1 = \left(x^2+y^2+(z-c)^2\right)^{1/2}, R_2 = \left(x^2+y^2+(z+c)^2\right)^{1/2}, R_3 = R_1 - z + c, R_4 = R_2 + z + c.$$

Expressions for $\hat{J}_3(t), \hat{J}_4(t)$ and $\hat{J}_5(t)$ for three viscoelastic models are given below:

For Kelvin Model

$$\hat{J}_3(t) = J_3\left(1-e^{-\frac{\mu+a}{\mu}T}\right),\ \hat{J}_4(t) = J_4\left(1-e^{-\frac{\mu+\mu_1}{\mu}T}\right), \hat{J}_5(t) = J_5\left(1-e^{-\frac{\mu+k\mu_1}{\mu}T}\right)$$

where $T = \frac{t}{t_1}$, t_1 is the relaxation time.

For Maxwell Model

$$\hat{J}_3(t)=J_3\left(1+\frac{\mu}{a}\left(1-e^{-\frac{a}{\mu+a}T}\right)\right),\ \hat{J}_4(t)=J_4\left(1+\frac{\mu}{\mu_1}\left(1-e^{-\frac{\mu_1}{\mu+\mu_1}T}\right)\right),$$

$$\hat{J}_5(t)=J_5\left(1+\frac{\mu}{k\mu_1}\left(1-e^{-\frac{k\mu_1}{\mu+k\mu_1}T}\right)\right)$$ where $T=\frac{t}{t_2}$, t_2 is the relaxation time.

For Standard Linear Solid Model

$$\hat{J}_3(t)=J_3\left(1+\frac{\mu}{\mu+2a}\left(1-e^{-\frac{\mu+2a}{\mu+a}\left(\frac{T}{2}\right)}\right)\right),\ \hat{J}_4(t)=J_4\left(1+\frac{\mu}{\mu+2\mu_1}\left(1-e^{-\frac{\mu+2\mu_1}{\mu+\mu_1}\left(\frac{T}{2}\right)}\right)\right),$$

$$\hat{J}_5(t)=J_5\left(1+\frac{\mu}{\mu+2k\mu_1}\left(1-e^{-\frac{\mu+2k\mu_1}{\mu+k\mu_1}\left(\frac{T}{2}\right)}\right)\right)$$ where $T=\frac{t}{t_2}$, t_2 is the relaxation time.

RESULTS AND DISCUSSION

To show the variation of elastic and viscoelastic displacements with the epicentral distance we assume that $\sigma_1=\sigma_2=\sigma$, $x=y=\frac{r}{\sqrt{2}}$ and define the dimensionless displacements as

$$\left(U_x,U_y,U_z\right)=\frac{4\pi\,\mu\,c}{F_0}\left(u_{(1)x},u_{(1)y},u_{(1)z}\right),\ z>0$$

$$=\frac{4\pi\,\mu\,c}{F_0}\left(u_{(2)x},u_{(2)y},u_{(2)z}\right),\ z<0$$

For graphical representation, we assume that $z=0, m=2$, $\kappa=2$ *and* $1\le T\le 3$.

Fig.2 shows the effect of the relaxation time on the displacement components of the horizontal dipole for the three viscoelastic models, namely, Kelvin, Maxwell, and Standard Linear Solid model. The values of displacement U_x and U_z along x-axis and z-axis increases with the increase of relaxation time. Graphical representation of displacement components for the three models is the same except for their maximum and minimum values.

The displacement components assume the maximum value in case of Maxwell and minimum in case of Kelvin for a particular value of time for all the displacements components as shown in table 1.

TABLE 1- Extremum. Values of the displacement components for HD (Horizontal dipole)

Models	Extremum. values of the displacement components for HD(Horizontal dipole)	
	Along x-axis	Along z-axis
	U_x	U_z
Kelvin	0.09	0.45
Maxwell	0.2	1
SLS	0.125	0.62

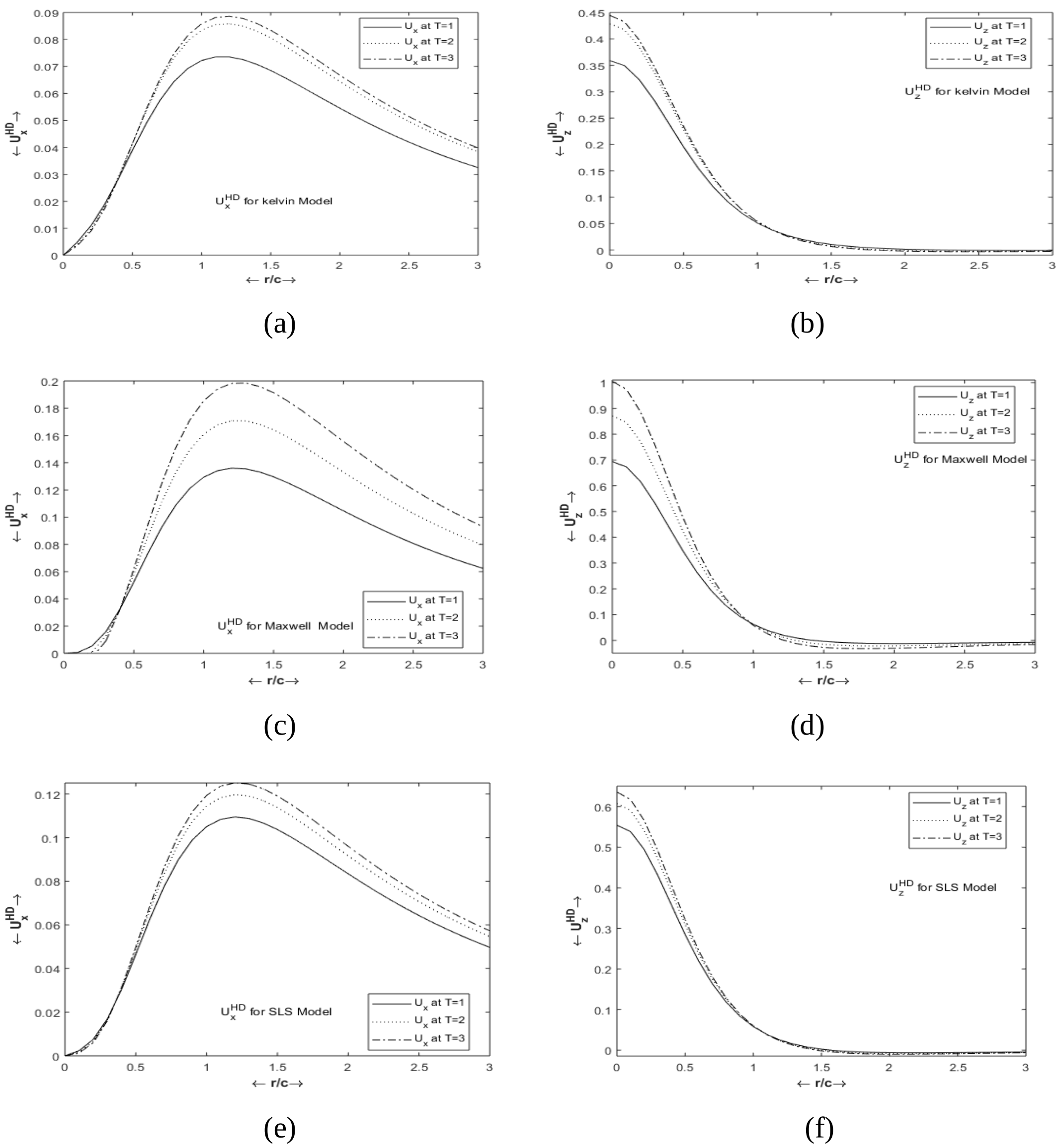


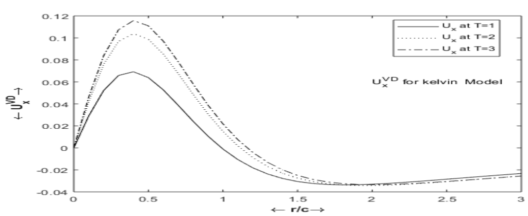
Fig 2. Effect of the relaxation time $1 \le T \le 3$ on the viscoelastic displacement field of the horizontal dipole at the interface $z = 0, m = 2$. (a, c, e) Horizontal displacement U_x for Kelvin Model, Maxwell Model, and SLS Model respectively, (b, d, e) Vertical displacement U_z for Kelvin Model, Maxwell Model, and SLS Model respectively with the epicentral distance r/c.

Fig.3 shows the effect of the relaxation time on the displacement components of the vertical dipole for the three viscoelastic models namely, Kelvin, Maxwell, and Standard Linear Solid model. The numerical values of displacement U_x and U_z along x-axis and z-axis increases with the increase of relaxation time. Graphical representation of displacement components for the three models is the same except for their maximum and minimum values.

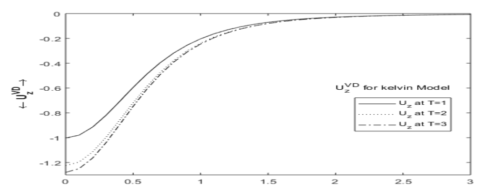
Numerically displacement components assume the maximum value in case of Maxwell and minimum in case of Kelvin for a particular value of time for all the displacements components as shown in table 2.

TABLE 2- Extremum. Values of the displacement components for VD (Vertical dipole)

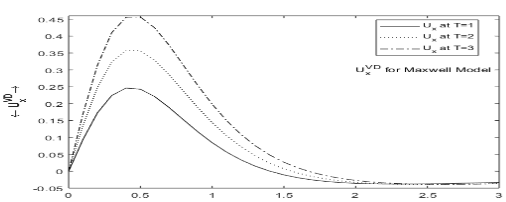
Models	Extremum. values of the displacement components for VD(Vertical dipole)	
	Along x-axis	Along z-axis
	U_x	U_z
Kelvin	0.12	-1.25
Maxwell	0.45	-3.1
SLS	0.22	-1.9



(a)



(b)



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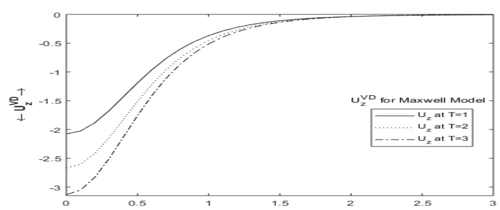

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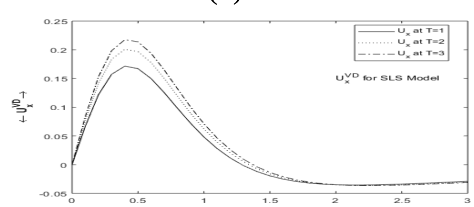
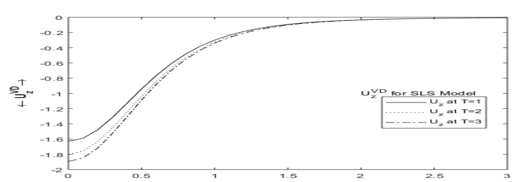


(e)



(f)

Fig 3. Effect of the relaxation time $1 \le T \le 3$ on the viscoelastic displacement field of the vertical dipole at the interface $z = 0, m = 2$. (a, c, e) Horizontal displacement U_x for Kelvin Model, Maxwell Model, and SLS Model respectively, (b, d, e) Vertical displacement U_z for Kelvin Model, Maxwell Model, and SLS Model respectively with the epicentral distance r/c.

CONCLUSION

The explicit expressions for the displacements in an elastic half-space overlying viscoelastic half-spaces due to the dipole source have been obtained. The results are also compared graphically for three viscoelastic models, namely, a Kelvin Model, a Maxwell Model, and a Standard linear solid Model (SLS). Graphical representations reveal that numerically the displacement components assume the maximum values in case of Maxwell Model and minimum values are obtained in case of Kelvin Model. Two welded half-spaces have the same elastic properties at $m = 1$. At $T = 0$ all the viscoelastic results are same as an elastic solution obtained by Kumari et al. (1992). This study may have possible applications in the field of geophysics and seismology.

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