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Square Multiplicative Labeling of Some Standard Graphs

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ABSTRACT

G is said to be a Square multiplicative labeling if there exists a bijection g:$V(G)\rightarrow\{1, 2, 3,....,p\}$ such that the induced function $g^*: E(G) \rightarrow N$ given by $g^*(uv) = [g(u)]^2. [g(v)]^2$ for every $uv \in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called Square multiplicative graph. In this paper, we show that the almost bipartite graph and ladder graph admit square multiplicative labeling.

KEYWORDS Almost bipartite graph, Labeling, Ladder graph, Square multiplicative labeling.

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INTRODUCTION

Labeling [4, 7] of graph G is the allocation of labels generally represented by integers to edges or vertices or both following certain conditions. There are several types of labeling techniques that have evolved from 1960s, one of the renouned labeling method is square multiplicative labeling[2,3,5]. In this paper we consider a simple, finite, connected and undirected graph.

Definition

G is said to be a *Square multiplicative labeling* if there exists a bijection $g:V(G)\to\{1,2,3,..,p\}$ such that the induced function $g^* : E(G) \to N$ given by $g^*(uv) = [g(u)]^2.[g(v)]^2$ for every $uv\in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called *Square multiplicative graph*.

Definition

An almost-bipartite graph[1] is a non-bipartite graph with the property that the removal of a particular single edge renders the graph bipartite.

Definition

The ladder graph[6] L_n $(n \geq 2)$ is the product graph $P_2\times P_n$ which contains $2n$ vertices and $3n$-2 edges.

RESULTS

***Theorem* 1 :** The Almost bipartite graph $P_m + e$ is square multiplicative.

Proof

Consider the graph $G = P_m + e$, where P_m is the path $u_1u_2u_3\ldots\ldots\ldots u_m$. Let V_1 and V_2 be the bipartition of the vertex set of G.

Case-1: When m is even, $e = u_1u_{m-1}$

$V_1 = \{u_1,u_3,\ldots,u_{m-1}\}$ and $V_2= \{u_2,u_4,\ldots..,u_m\}$.

Case-2: When m is odd, $e = u_1u_m$

$V_1 = \{u_1,u_3,\ldots,u_m\}$ and $V_2= \{u_2,u_4,\ldots..,u_{m-1}\}$.

For both the cases we define $g:V(G)\to\{1, 2, 3, \ldots\ldots, m\}$ by $g(u_r) = r, 1 \leq r \leq m$.

The function g induces a square multiplicative labeling on G.

For if, g^* be the induced function defined by $g^* : E \to N$ such that $g^*(u_r u_s) = r^2s^2$

Case-1: When m is even, $e = u_1u_{m-1}$

Let $E = E_1\cup E_2\cup E_3$ where,

$E_1=\left\{e_r \middle| e_r = u_{2r-1}u_{2r}, 1 \leq r \leq \frac{m}{2}\right\}$,

$E_2=\left\{e_r \middle| e_r = u_{2r}u_{2r+1}, 1 \leq r \leq \frac{m}{2} - 1\right\}$,

$E_3 = \{e = u_1 u_{m-1}\}$.

To prove that $g^*$is injective in E.

Claim 1 :g^* is injective inE_1.

Let e_r, $e_s \in E_1$

$g^*(e_r) = g^*(u_{2r-1} u_{2r})$

$= [g(u_{2r-1})]^2 [g(u_{2r})]^2$

$= (2r-1)^2(2r)^2$

$g^*(e_r) = 2^2 r^2 (2r-1)^2$

$g^*(e_s) = g^*(u_{2s-1} u_{2s})$

$= [g(u_{2s-1})]^2 [g(u_{2s})]^2$

$= (2s-1)^2(2s)^2$

$g^*(e_s) = 2^2 s^2 (2s-1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_1.

We find that all the labeling of edges in E_1 are multiples of 2^2 .

Claim 2 :g^* is injective inE_2.

Let e_r, $e_s \in E_2$

$g^*(e_r) = g^*(u_{2r} u_{2r+1})$

$= [g(u_{2r})]^2 [g(u_{2r+1})]^2$

$= (2r)^2 (2r+1)^2$

$g^*(e_r) = 2^2 r^2 (2r+1)^2$

$g^*(e_s) = g^*(u_{2s} u_{2s+1})$

$= [g(u_{2s})]^2 [g(u_{2s+1})]^2$

$= (2s)^2 (2s+1)^2$

$g^*(e_s) = 2^2 s^2 (2s+1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_2.

We observe that all the labelings of edges in E_2 are multiples of 2^2 .

Claim 3 :g^* is injective in E_3.

Let $e \in E_3$

$g^*(e) = g^*(u_1 u_{m-1})$

$= [g(u_1)]^2 [g(u_{m-1})]^2$

$g^*(e) = (1)^2 (m-1)^2$

Since we have only one edge in E_3, g^* is injective in E_3.

***Claim 4*:** g^* is injective among E_1, E_2 and E_3 .

We notice that all the labeling of edges in E_1 and E_2 are multiple of 2^2 and the edge label of E_3 is odd. Hence it is obvious that all the labelings of edges of E_1 and E_2 are distinct from the edge label of E_3 . Now we have to show that labelings of edges in E_1 and E_2 are distinct.

***Claim 4.1*:** g^* is injective among E_1 and E_2 .

Let $e_r \in E_1$, $e_s \in E_2$

$g^*(e_r) = g^*(u_{2r-1}\, u_{2r})$

$= [g(u_{2r-1})]^2\, [g(u_{2r})]^2$

$= (2r-1)^2(2r)^2$

$g^*(e_r) = 2^2r^2\,(2r-1)^2$

$g^*(e_s) = g^*(u_{2s}u_{2s+1})$

$= [g(u_{2s})]^2\, [g(u_{2s+1})]^2$

$= (2s)^2\,(2s+1)^2$

$g^*(e_s) = 2^2s^2\,(2s+1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_1 and E_2.

$\Rightarrow$ All the edge labels in E are distinct ,when m is even.

Hence G admits square multiplicative labeling when m is even.

Hence G is a Square multiplicative Graph when m is even .

Case-2:When m is odd ,$e = u_1u_m$

Let $E = E_1 \cup E_2 \cup E_3$ where,

$E_1 = \left\{ e_r \middle| \, e_r = u_{2r-1}\, u_{2r} \,, 1 \leq r \leq \frac{m+1}{2} - 1 \right\}$,

$E_2 = \left\{ e_r \middle| \, e_r = u_{2r}u_{2r+1} \,, 1 \leq r \leq \frac{m+1}{2} - 1 \right\}$,

$E_3 = \{e = u_1u_m\}$.

To prove that g^* is injective in E.

Claim 1 **:** g^* is injective in E_1.

Let $e_r, e_s \in E_1$

$g^*(e_r) = g^*(u_{2r-1}\, u_{2r})$

$= [g(u_{2r-1})]^2\, [g(u_{2r})]^2$

$= (2r-1)^2(2r)^2$

$g^*(e_r) = 2^2r^2\,(2r-1)^2$

$g^*(e_s) = g^*(u_{2s-1}\, u_{2s})$

$= [g(u_{2s-1})]^2 [g(u_{2s})]^2$

$= (2s-1)^2(2s)^2$

$g^*(e_s) = 2^2s^2 (2s-1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_1.

We find that all the labeling of edges in E_1 are multiples of 2^2 .

Claim 2 : g^* is injective in E_2.

Let $e_r, e_s \in E_2$

$g^*(e_r) = g^*(u_{2r}u_{2r+1})$

$= [g(u_{2r})]^2 [g(u_{2r+1})]^2$

$= (2r)^2 (2r+1)^2$

$g^*(e_r) = 2^2r^2 (2r+1)^2$

$g^*(e_s) = g^*(u_{2s}u_{2s+1})$

$= [g(u_{2s})]^2 [g(u_{2s+1})]^2$

$= (2s)^2 (2s+1)^2$

$g^*(e_s) = 2^2s^2 (2s+1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$. Hence g^* is injective in E_2.

We observe that all the labelings of edges in E_2 are multiples of 2^2 .

Claim 3 : g^* is injective in E_3.

Let $e \in E_3$

$g^*(e) = g^*(u_1u_m)$

$= [g(u_1)]^2 [g(u_m)]^2$

$g^*(e) = (1)^2 (m)^2$

Since we have only one edge in E_3 , g^* is injective in E_3.

Claim 4 : g^* is injective among E_1, E_2 and E_3 .

We notice that all the labeling of edges in E_1 and E_2 are multiple of 2^2 and the edge label of E_3 is odd. Hence it is obvious that all the labelings of edges of E_1 and E_2 are distinct from the edge label of E_3 . Now we have to show that labelings of edges in E_1 and E_2 are distinct.

Claim 4.1: g^* is injective among E_1 and E_2 .

Let $e_r \in E_1$, $e_s \in E_2$

$g^*(e_r) = g^*(u_{2r-1}u_{2r})$

$= [g(u_{2r-1})]^2 [g(u_{2r})]^2$

$= (2r-1)^2(2r)^2$

$g^*(e_r) = 2^2 r^2 (2r - 1)^2$

$g^*(e_s) = g^*(u_{2s} u_{2s+1})$

$= [g(u_{2s})]^2 [g(u_{2s+1})]^2$

$= (2s)^2 (2s + 1)^2$

$g^*(e_s) = 2^2 s^2 (2s + 1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_1 and E_2.

$\Rightarrow$ All the edge labels in E are distinct ,when m is odd .

Hence G admits square multiplicative labeling when m is odd.

$\Rightarrow G$ admits Square multiplicative labelingfor both the cases .

Hence G is a Square multiplicative Graph .

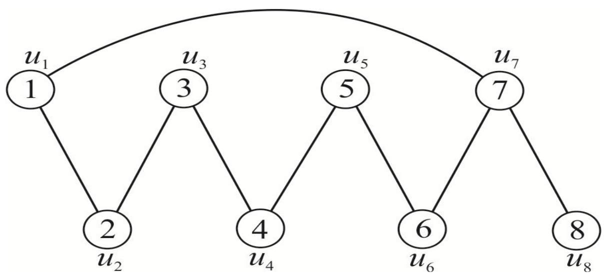

Figure 1: Square Multiplicative Labeling Of Almost Bipartite Graph $P_8 + e$.

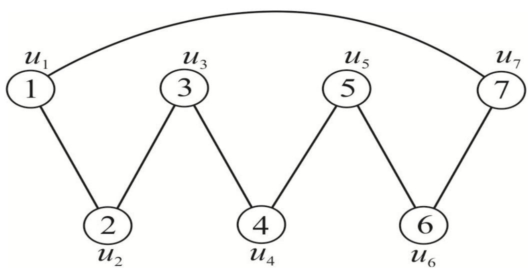

Figure 2: Square Multiplicative Labeling Of Almost Bipartite Graph $P_7 + e$.

***Theorem* 2:** The Ladder graph L_n is square multiplicative.

***Proof* :**

We have $|V(L_n)| = 2n$ and $|E(L_n)| = 3n - 2$.

Let V_1 and V_2 be the bipartition of the vertex set V where

$V_1 = \{ v_1, v_2, \ldots . v_n \}$ and $V_2 = \{ u_1, u_2, \ldots .., u_n \}$.

Let $E = E_1 \cup E_2 \cup E_3$ where,

$E_1 = \{e_r | \ e_r = v_r v_{r+1}, 1 \leq r \leq n - 1 \}$,

$E_2 = \{e_r | \ e_r = u_r u_{r+1}, 1 \leq r \leq n - 1 \}$,

$E_3 = \{e_r | \ e_r = v_r u_r, 1 \leq r \leq n \}$.

Define $g: V(G) \rightarrow \{1, 2, 3, \ldots \ldots, 2n\}$ by

$g(v_r) = 2r - 1, 1 \le r \le n,$

$g(u_r) = 2r, 1 \le r \le n$.

The function g induces a square multiplicative labeling on G.

For if, g^* be the induced function defined by $g^* : E \rightarrow N$ such that

$g^*(v_r\, u_s) = (2r - 1)^2(2s)^2$

To prove that $g^*$is injective in E.

Claim 1 : g^* is injective in E_1.

Let $e_r, e_s \in E_1$

$g^*(e_r) = g^*(v_r v_{r+1})$

$\quad = [g(v_r)]^2 [g(v_{r+1})]^2$

$g^*(e_r) = (2r - 1)^2(2r + 1)^2$

$g^*(e_s) = g^*(v_s v_{s+1})$

$\quad = [g(v_s)]^2 [g(v_{s+1})]^2$

$g^*(e_s) = (2s - 1)^2(2s + 1)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_1

Claim 2 :g^* is injective in E_2.

Let $e_r, e_s \in E_1$

$g^*(e_r) = g^*(u_r u_{r+1})$

$\quad = [g(u_r)]^2 [g(u_{r+1})]^2$

$g^*(e_r) = (2r)^2(2(r + 1))^2$

$g^*(e_r) = (2)^4(r(r + 1))^2$

$g^*(e_s) = g^*(u_s u_{s+1})$

$\quad = [g(u_s)]^2 [g(u_{s+1})]^2$

$g^*(e_r) = (2s)^2(2(s + 1))^2$

$g^*(e_r) = (2)^4(s(s + 1))^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_2

We note that all the labelings of edges in E_2 are multiples of 2^4

Claim 3 :g^* is injective in E_3.

Let $e_r, e_s \in E_3$

$g^*(e_r) = g^*(v_r u_r)$

$\quad = [g(v_r)]^2 [g(u_r)]^2$

$= (2r-1)^2(2r)^2$

$g^*(e_r) = (2)^2\,(2r-1)^2(r)^2$

$g^*(e_s) = g^*(v_s u_s)$

$= [g(v_s)]^2\,[g(u_s)]^2$

$= (2s-1)^2(2s)^2$

$g^*(e_s) = (2)^2(2s-1)^2(s)^2$

Hence for $r \neq s$, $g^*(e_r) \neq g^*(e_s)$

Hence g^* is injective in E_3

We note that all the labelings of edges in E_3 are multiples of 2^2.

Claim 4 : g^* is injective among E_1, E_2 and E_3 .

We note that all the labelings of edges in E_2 are multiples of 2^4.

Hence it is very clear that all the edge labels of E_2 are distinct from the edge labels of E_1 and E_3

Also we find that the edge labels of E_3 are are multiples of 2^2.

Hence all the edge labels of E_3 are distinct from the edge labels of E_1

so all the edge labels in E are distinct .

Hence L_n admits square multiplicative labeling

$\therefore$ The ladder graph L_n is square multiplicative .

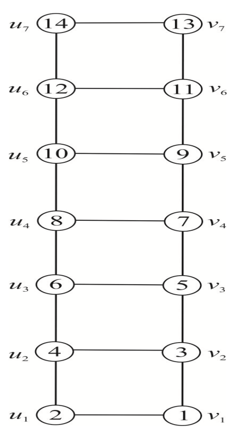

Figure 3:Square Multiplicative Labeling Of Ladder Graph L_7

CONCLUSION

In this paper, the admittance of square multiplicative labeling to a variety of graphs namely almost bipartite graph and ladder graph has been discussed.

For almost bipartite graph two cases are discussed.This concept can be extended for different non-regular graphs.

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