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t- Pebbling number of a class of diameter two Graphs

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ABSTRACT

For a graph G , define the pebbling number $f(G, v)$ to be the smallest integer m such that, for any distribution of m pebbles to the vertices of G , one pebble can be moved to a specified target vertex v and $f(G) = \max_{v \in V(G)} f(G, v)$. The t -pebbling number $f_t(G)$ to be smallest integer in

such that, for any distribution of m such that, for any distribution of m pebbles to the vertices of G , t pebbles can be moved to any specified target vertex. Pebbling number of large classes of graphs such as trees diameter two graphs, class zero graph etc. are calculated and is in Hulbert². t -Pebbling number of graphs is another attraction in this area. The t -pebbling number of different kinds of graph and bounce two certain classes of graphs were discovered and is also seen in Hulbert². This paper contains the generalized t -pebbling number of a class of diameter two graphs.

KEYWORDS: Pebbling, t -pebbling, diameter two graphs, pebbling number AMS Subject Classification 05C99

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INTRODUCTION

The concept of pebbling in graphs² arose from an attempt by Lagarias and Saks to give an alternative proof of a theorem of Lemke and Kleitman. It is known that for any set $N = \{n_1, n_2, \dots, n_q\}$ of q natural numbers, there is a nonempty index set $I \subset \{1, 2, \dots, q\}$ such that

$$q \mid \sum_{i \in I} n_i \quad \text{and} \quad \sum_{i \in I} \gcd(q, n_i) \leq q$$

However the proof was very complicated. It was the intention of Lagarias and Saks to introduce Graph pebbling as a more intuitive vehicle for providing the theorem.

The purpose was accomplished by FRL Chung¹ in 1989 when she published the first paper in graph pebbling.

2. Graph Pebbling

Suppose D is a distribution in which P pebbles are distributed on to the vertices of a graph G . Let $p(v)$ denotes the number of pebbles on the vertex v . A pebbling step consists of removing two pebbles from one vertex and then placing one pebble on to an adjacent vertex. We say a pebble can be moved to r , the ‘target vertex’, if we can apply pebbling steps repeatedly so that in the resulting distribution, r has a pebble on it.

Definition: Pebbling number¹. For a graph G , we define the pebbling number $f(G, v)$ to be the smallest integer m such that, for any distribution of m pebbles to the vertices of G , one pebble can be moved to a specified target vertex $= v$. Again,

$$f(G) = \max_{v \in V(G)} f(G, v)$$

Definition¹ t -pebbling number: For a graph G , let $f_t(G, v)$ denote the smallest integer m such that, if m pebbles are assigned to the vertices of G , then t pebbles can be moved to V . Again

$$f_t(G) = \max_{v \in V(G)} f_t(G, v)$$

3. t – pebbling number of a class of diameter two graphs.

Pachter et al³ showed that, for any diameter two graph G , $f(G) = n$ or $n+1$, where n is the number of vertices in G .

In this paper, we completely characterize the t – pebbling number of a large class of diameter two graphs.

Lemma 1: Let u be any vertex of G of degree at least two. Then

$$f_t(G, u) \geq 4$$

Equality holds of $\deg u = n - 1$

Proof: Let $v \in N[u]$

If $p(u) = 2t-1, p(v) = 0, p(w) = 1, \forall w \in V(G) - \{u, v\}$ then t pebbles can not be place on to v .

Therefore, $f_t(G, V) = 2t + n - 2$.

Lemma 2: Let v be any vertex of G of degree less than $n - 1$. Then $f_t(G, v) \geq 4t$

Proof:- Let $u \notin N[v]$

If $p(v) = 4t - 1$, then t pebbles cannot be moved to u .

Hence $f_t(G, v) \geq 4t$

Corollary 3: For any vertex v of degree less than $n - 1$, we have $f_t(G, v) \geq \max(2t+n-2, 4t)$

We next discuss a case when equality holds in the above inequality.

Theorem 4: Let u and v be non adjacent vertices of G such that $\deg u = \deg v = n - 2$. Then

$f_t(G, v) = f_t(G, u) = \max(4t, 2t + n - 2)$.

Proof: When $t = 1$, the result reads as $f(G, v) = \max(4, n)$.

If $n = 3$, then $G \cong P_3$, the path of three vertices. Also u and v are end vertices of G . It is known that $f(G, v) = 4$.

If $n \geq 4$, the result reads as $f(G, v) = n$. Let D be a distribution of n pebbles on the vertices of G . If $p(v) > 0$ or $p(\omega) \geq 2$ for any vertex $\omega \in V(G) - \{u, v\}$ the result is proved. So, we may assume $p(v) = 0, P(\omega) \leq 1, \forall \omega \in V(G) - \{u, v\}$

This implies $p(v) \geq 2$

If $p(\omega) \geq 1$ for at least one $\omega \in V(G) - \{u, v\}$ v can be pebbled using the path $u \omega v$

Otherwise $p(x) = 0, \forall x \in V(G) - \{u, v\}$

This implies $p(u) = n \geq 4$

Therefore v can be pebbled. Thus the result is verified for $t = 1$

Let $V_1 = V - \{u, v\} = \{u_1, u_2, \dots, u_{n-2}\}$

Case (a): $2t \leq n - 2$

In this case, we have to prove that $f_t(G, v) = 2t + (n-2)$. We prove this by induction on t . Assume that the result is true for $1, 2, \dots, t - 1$. Let D be the distribution of $s = 2t + n - 2$ pebbles on G . If either $p(v) \geq 1$ or $p(x) \geq 2$ for any $x \in V_1$, we can move on pebble to v at the cost of at most

two pebbles By induction we can move $(t - 1)$ more pebbles to v . Therefore we may assume $p(v) = 0, p(x) \leq 1 \forall x \in V_1$. Let $K = \{x / x \in V_1, p(x) = 1\}$ Therefore $p(u) = s - k$.

Case (1):

In this case $(s - k) = 2t + (n - 2) - k \geq 2t$. We can move one pebble each to t vertices in V_1 . Then we can move $t - k$ pebbles to v .

Case (2): $K < t$.

In this case if we can move $2t - k$ pebbles to V_1 , We are through.

We have $(s - k) \geq 4t - 2k$

We can move the pebbles in such a way that each of the $k -$ occupied vertices have an even number of pebbles, the total number being $2t$. Thus, we can move t pebbles to v . Therefore the result is true by induction for all t . such that $2t \leq n - 2$.

Case (b): $2t = (n - 2) + 1$.

In this case we prove $f_t(G, v) \leq 4t$. Let D be a distribution of $4t$ pebbles on G . Here $2(t - 1) = (n - 2) - 1$. By case (s),

$F_{t-1}(G, v) = 4t - 3$.

Clearly, we may assume $p(v) = 0$. Consider the following cases:

(i) $p(v) \geq 1$ (ii) $p(u) \geq 2$ and $p(w) \geq 1$ for some $w \in V_1$ (iii) $p(w) \geq 2$ for some $w \in V_1$

In each of the above cases, we can move one pebble to v (this includes case (i) also) at the cost of at most three pebbles: That is, after a pebbling process, we will have one pebble at v and at least $(4t - 3)$ pebbles on $V(G)$ [Excluding the one pebble at V]. The by induction,

We can move $(t - 1)$ pebbles to v , making t pebbles in all at v .

Thus we may assume $p(v) = 0, p(w) \leq 1 \forall w \in V_1$

Then $p(u) \geq 4t - (n - 2) \geq 2t + 1$

If now $p(w) \geq 1$ for some $w \in V_1$ we have proved (case (iii) above). Thus we may assume

$\sum_{w \in V_1} P(w) = 0$, implying $p(u) = 4t$ enabling us to move t pebbles to v .

Case (c) $2t \geq n$.

Again, we prove by induction on t . Whatever be the distribution of $4t$ pebbles on V , we can move one pebble to v at the cost of at most 4 pebbles. The remaining number of pebbles will be $4t - 4$, with $2(2t - 1) \geq n - 2$. Therefore the result is true by induction.

Hence the proof of the theorem.

Corollary 5: $f_t(G) = 4t$ if and only if either $2t \geq n - 2$ and the minimum degree of all the vertices in G is $n - 2$ or G is complete and $2t = n - 2$.

Proof: First, suppose $ft(G) = 4t$. Lemma 1 implies $2t \geq n - 2$.

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