

## *International Journal of Scientific Research and Reviews*

### **Variance of Time to Recruitment with A Record of N Decision**

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#### **ABSTRACT**

In any graded marketing system the shortage of manpower takes place due to policy decision implemented by the organization, firing of job, dismissal etc. This shortage of manpower causes depletion of manpower. In realistic, every policy decision does not lead to attrition of manpower, which leads to consider only the policy which produces shortage of manpower with a probability. Hence from  $n+r$  policy implemented by the organization, only  $n$  policy are considered which produces shortage of manpower. In this paper two stochastic models are constructed by assuming that shortage of manpower and inter-policy decision times form a two different sequences of independent and identically distributed random variables with two different cases of breakdown threshold. The analytical results are derived using a univariate CUM policy of recruitment. The analytical results are numerically illustrated and the appropriate conclusion are presented.

**KEYWORDS:** Single graded marketing organization inter-policy decision times, univariate CUM policy.

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**AMS SUBJECT CLASSIFICATION:**primary: 90B70, secondary 60H30, 60K05.

## **INTRODUCTION**

Abrasion of manpower leads to diminution of manpower which is quite common in any marketing organization. Whenever the marketing organization introduces a policy, shortage of manpower takes place only for some policy, so recruitment has to take place to overcome this shortage. But frequent recruitment is not desirable hence the recruitment is done based on the shock model approach. Many renewal theory models on manpower system have been studied by<sup>1,2</sup> to fill up the gap in any manpower system. has derived the analytical results for the variance of time to recruitment using univariate CUM policy by considering that every policy produces shortage of manpower when the breakdown threshold has one or more components. It is not pragmatic that attrition takes place for every policy made by the organization. In this context the problem of time to requirement for single grade manpower system is framed in a way that, the policy decision which produces the shortage of manpower are considered with probability  $p$  ( $0 \leq p < 1$ ). Two stochastic models are constructed by assuming that shortage of manpower and inter policy decisions times forms a two different sequence of independent and identical distributed exponential random variables. In the first model the breakdown threshold has a single component for the cumulative shortage of manpower. For the second model the breakdown threshold has two components. The first component arises for the cumulative shortage of manpower and the second component arises due to the cumulative breaks taken by the employee in the organization. The variance of time to recruitment is derived for two stochastic models using univariate CUM policy of recruitment. The realistic conclusion which is made by the numerical illustration supports the analytical results.

## **MODEL DESCRIPTION**

A single grade manpower system in which the policy decision that produces shortage of manpower are considered. Let  $D_i$ ,  $i=1,2,3$  be a stochastic process denoting that shortage of manpower due to  $i^{\text{th}}$  policy decision with distribution function  $A(\cdot)$ , density function  $a(\cdot)$  and Laplace transform  $\bar{a}(\cdot)$  with mean  $\frac{1}{\gamma}$ ,  $\gamma > 0$ . Let  $S_{D_i}$  be the cumulative shortage of manpower upto  $i$  policy decision with distributive function  $A_i(\cdot)$ , density function  $a_i(\cdot)$  and Laplace transform  $\bar{a}_i(\cdot)$ . Let  $E_i$ ,  $i=1,2,3$  is a stochastic process denoting that time between  $(i-1)^{\text{th}}$  and  $i^{\text{th}}$  policy decision that produces shortage of manpower with distributive function  $B(\cdot)$ , density function  $b(\cdot)$  and Laplace transform  $\bar{b}(\cdot)$  with mean  $\frac{1}{\eta}$ ,  $\eta > 0$ . Let  $S_{E_i}$ ,  $i=1,2,3$  be a waiting time upto  $i$  policy decision with distributive function  $B_i(\cdot)$ , density function  $b_i(\cdot)$  and the Laplace transform  $\bar{b}_i(\cdot)$ . Let  $N(t)$  denotes

the number of policy in  $[0,t)$  and  $M(t)$  denotes the number of policy that produces shortage of manpower with probability  $p$ , ( $0 \leq p < 1$ ). Let  $S_{D_{M(t)}}$  represents cumulative shortage of manpower upto  $M(t)$  policy decision. Let the exponential random variable  $W_1$  represents the threshold for the cumulative shortage of manpower with parameter  $\tau_1 > 0$  and the exponential random variable  $W_2$  represents the threshold for the cumulative breaks taken by the employee in the organization with parameter  $\tau_2, \tau_2 > 0$ . Let the random variable  $R$  represents the time to recruitment in the organization with distributive function  $K(\cdot)$ , density function  $k(\cdot)$  and laplace transform  $\bar{k}(\cdot)$ . Recruitment is done using the recruitment policy based on shock model approach which is denoted as univariate CUM policy of recruitment. It is defined as the recruitment is done when the cumulative shortage of manpower exceeds the breakdown threshold. It is assumed that shortage of manpower, inter-policy decision times and the breakdown threshold are assumed to be statistically independent.

## **ANALYTICAL RESULTS**

### **Model-I**

The event of time to recruitment is defined as follows: The event recruitment occurs beyond the time  $t$  is equivalent to the cumulative shortage of manpower upto  $M(t)$  policy that does not exceeds the breakdown threshold upto the time  $t, t > 0$ . Since the two events are equal which implies their probabilities for the occurrence are equal. It is given by

$$P(R > t) \Leftrightarrow P(S_{D_{M(t)}} < W).$$

By conditioning upon  $M(t)$ ,  $P(R > t) = \sum_{n=0}^{\infty} P(S_{D_n} < W) P(M(t) = n)$ .

Now conditioning on  $S_{D_n}$  and by further simplification

$$P(S_{D_n} < W) = \int_0^{\infty} P(W > t) dP(S_{D_n} < t) = (\bar{a}(\tau_1))^n$$

Since, the inter-policy decision times follows a sequence of independent and identically distributed exponential random variables, the number of policy decisions  $N(t)$ , such that  $P(N(t) = n)$  follows a Poisson process with mean  $\eta t$ . Considering only the decisions that produces shortage of manpower with the probability  $p$  is realistic.

Let  $Q_r = (n+r)$  policy decisions occur in  $[0,t)$  and exactly  $n$  policy decisions are recorded out of  $(n+r)$ .

$$P(M(t) = n) = \sum_{r=0}^{\infty} P(Q_r) = \sum_{r=0}^{\infty} \frac{e^{-\eta t} (\eta t)^{n+r}}{(n+r)!} (n+r) c_r p^n q^r = \frac{e^{-\eta p t} (\eta p t)^n}{n!}.$$

Using this result the distribution function of time to recruitment is given by,  $K(t) = 1 - e^{-\left(\frac{\eta p \tau}{\gamma + \tau_1}\right)t}$

Now, taking the Laplace transform and deriving the first moment by differentiating the Laplace transform for the probability density function of the random variable R at  $s=0$  gives the mean time to recruitment as follows.

$$E(R) = \frac{\gamma + \tau}{\eta p \tau_1}$$

The second moment of the random variable R is determined by differentiating twice the Laplace transform of R at  $s=0$ . From the two moments, the variance of time to recruitment is determined for the present model. It is given as

$$V(R) = \left(\frac{\gamma + \tau}{\eta p \tau_1}\right)^2$$

**Model-II:**

Proceeding as in Model-I, the probability of the event, recruitment occurs beyond the time  $t > 0$  is given as  $P(R > t) \Leftrightarrow P(S_{D_{M(t)}} < W_1 + W_2)$ . Now conditioning upon  $M(t)$  and  $S_{D_{M(t)}}$ , the distribution of the random variable R is determined, it is given by

$$K(t) = 1 - \frac{\tau_2}{\tau_2 - \tau_1} e^{-\eta p (1 - \bar{a}(\tau_1)) t} + \frac{\tau_1}{\tau_2 - \tau_1} e^{-\eta p (1 - \bar{a}(\tau_2)) t}$$

Hence the moments of the random variable R is determined by differentiating the Laplace transform of time to recruitment at  $s=0$ .

Hence, the first and second moment of time to recruitment is given by

$$E(R) = \frac{1}{\eta p (\tau_2 - \tau_1)} \left( \frac{\tau_2 (\gamma + \tau_1)}{\tau_1} - \frac{\tau_1 (\gamma + \tau_2)}{\tau_2} \right)$$

$$E(R^2) = \frac{2}{(\tau_2 - \tau_1) (\eta p)^2} \left( \frac{\tau_2 (\gamma + \tau_1)^2}{\tau_1} - \frac{\tau_1 (\gamma + \tau_2)^2}{\tau_2} \right)$$

From the two moments, the variance of time to recruitment is determined.

**NUMERICAL ILLUSTRATION**

The effect of the parameters involved in the analytical results are studied for the two stochastic models and the conclusion has been made by analyzing the results derived by the mean and variance of time to recruitment.

### Model-I

Effect of  $\eta, \rho, \gamma$  for the mean and variance of time to recruitment is studied by fixing the value of the parameter.

| $\tau_1$ | $\eta$ | $\gamma$ | $\rho$ | E(R)   | V(R)    |
|----------|--------|----------|--------|--------|---------|
| 0.1      | 0.7    | 0.01     | 0.9    | 1.7460 | 3.30486 |
| 0.1      | 0.8    | 0.01     | 0.9    | 1.5278 | 2.3341  |
| 0.1      | 0.9    | 0.01     | 0.9    | 1.3580 | 1.8442  |
| 0.1      | 0.6    | 0.02     | 0.4    | 2.2222 | 4.9383  |
| 0.1      | 0.6    | 0.03     | 0.4    | 2.4074 | 5.7956  |
| 0.1      | 0.6    | 0.04     | 0.4    | 2.5926 | 6.7215  |
| 0.1      | 0.6    | 0.01     | 0.9    | 2.0370 | 4.1495  |
| 0.1      | 0.6    | 0.01     | 0.92   | 1.9928 | 3.9711  |
| 0.1      | 0.6    | 0.01     | 0.94   | 1.9504 | 3.8039  |

### Model-II

Effect of  $\gamma, \eta, \rho$  for the mean and variance of time to recruitment is studied by fixing the value of the parameters

| $\tau_1$ | $\tau_2$ | $\eta$ | $\gamma$ | P   | E(R)   | V(R)   |
|----------|----------|--------|----------|-----|--------|--------|
| 0.8      | 0.85     | 0.6    | 0.3      | 0.6 | 4.7998 | 20.996 |
| 0.8      | 0.85     | 0.7    | 0.3      | 0.6 | 4.1141 | 15.426 |
| 0.8      | 0.85     | 0.8    | 0.3      | 0.6 | 3.5999 | 11.810 |
| 0.8      | 0.85     | 0.5    | 0.4      | 0.6 | 6.5686 | 37.918 |
| 0.8      | 0.85     | 0.5    | 0.5      | 0.6 | 7.3775 | 46.257 |
| 0.8      | 0.85     | 0.5    | 0.6      | 0.6 | 8.1863 | 55.250 |
| 0.8      | 0.85     | 0.5    | 0.3      | 0.7 | 4.9370 | 22.213 |
| 0.8      | 0.85     | 0.5    | 0.3      | 0.8 | 4.3199 | 17.007 |
| 0.8      | 0.85     | 0.5    | 0.3      | 0.9 | 3.8399 | 13.437 |

### CONCLUSION

The numerical results derived in the two models coincides with the realistic supposition. In Model-I and II, when the time for making decisions increases, then the exceeding of the threshold for the cumulative shortage of manpower produced due to decisions will be elongated. This lead to the delay of recruitment and the time to recruitment increases. If the cumulative shortage of manpower increases, the mean time to recruitment decreases. This numerical conclusion coincides with realistic.

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