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Antiflexible Derivation Alternator Rings

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ABSTRACT:

Kleinfeld defined two different generalizations of alternative rings, and for each of these generalizations he proved that the simple rings are alternative. Both these generalizations defined by Kleinfeld are contained in the varieties of derivation alternator rings. These derivation alternator rings were initially studied by Hentzel et. all. Hentzel and Smith investigated the structure of non-associative, flexible derivation rings and Nimmo investigated structure of non-associative anti-commutative derivation alternator rings.

In the study of non-associative rings one of the important class of rings is derivation alternator rings. In this paper, we proved that antiflexible derivation alternator ring under some conditions is commutative or alternative.

KEYWORDS: Derivation alternator ring, Antiflexible ring, Alternative ring, Characteristic, Commutator.

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PRELIMINARIES:

A non-associative ring with characteristic $\neq 2$ is called a derivation alternator ring if it satisfies the identities

$$(x,x,x) = 0 \quad (1)$$

$$(yz,x,x) = y(z,x,x) + (y,x,x)z \quad (2)$$

$$(x,x,yz) = y(x,x,z) + (x,x,y)z \quad (3)$$

The structure of non-associative, antiflexible rings that satisfy equations (1) to (3). We note that antiflexible derivation alternator rings can be defined simply by equations (1) and (2) and the identity

$$A(x,y,z) = (x,y,z) - (z,y,x) = 0 \quad (4)$$

Throught this section R will denote antiflexible derivation alternator ring of characteristic $\neq 2$.

From eq. (1) and (4), we have

$$J(x,y,z) = (x,y,z) + (y,z,x) + (z,x,y) = 0 \quad (5)$$

From equations (1) to (3), we have

$$(x,yz,x) = y(x,z,x) + (x,y,x)z \quad (6)$$

By linearizing (6), we get

$$(x,yz,w) = y(x,z,w) + (x,y,w)z \quad (7)$$

Put $z = y$ in (6) which gives

$$(x,y^2,x) = y(x,y,x) + (x,y,x)y$$

$$= yo(x,y,x) \quad (8)$$

for $a0b = ab + ba$ the Jordan product of the elements.

We have the identity valid in any ring the so called Teichmuller identity

$$C(w,x,y,z) = (wx,y,z) - (w,xy,z) + (w,x,yz) - w(x,y,z) - (w,x,y)z = 0$$

(9)

The following identities hold in any ring

$$G(x,y,z) = (xy,z) - x(y,z) - (x,y)z - (x,y,z) + (x,z,y) - (z,x,y) = 0 \quad (10)$$

Since, in any ring

$$(xo\,y)oz - xo(yo\,z) = (x,y,z) + (x,z,y) + (y,x,z) - (y,z,x) - (z,x,y) - (z,y,x) + (y,(x,z))$$

Then following identity is employed often:

$$(xoy)oz - xo(yo\,z) = (y,(x,z)) \quad (11)$$

So the following identity holds in any ring,

$$(xo\,y)oz - xo(yo\,z) = (x,y,z) + (x,z,y) + (y,x,z) - (y,z,x) - (z,x,y) - (z,y,x) + (y,(x,z))$$

So that from eq. (5), we get

$(xo\,y,z)+(yo\,z,x)+(zo\,x,y)=0.$ (12)

Forming $0=C(w,x,y,z)-C(x,y,z,w)+C(y,z,w,x)-C(z,w,xy)$

And using equation (5), we obtain,

$K(w,x,y,z)=(w,(x,y,z))-(x,(y,z,w))+(y,(z,w,x))-(z,(w,x,y))=0.$ (13)

As Anderson and Outcalt [6], expanding

$$0=C(w,x,y,z)-C(z,y,x,w)+C(x,y,z,w)-C(w,z,y,x)+C(y,z,w,x)-C(x,w,z,y)-C(y,x,w,z)$$
$$-J((w,x),y,z)-J((x,y),z,w)-J((y,z),w,x)-J((z,w),x,y)$$

and using equation(4),we get

$H(w,x,y,z)=(w,(x,y),z)+(x,(y,z),w)+(y,(z,w),x)+(z,(w,x),y)=0$ (14)

From $K(x,x,x,y)+(x,J(x,y,x))=0.$

It follows that $2(x,(x,y,x))=0$.

Therefore $\big(x,(x,y,x)\big)=0.$ (15)

From this and the fact that $(x,y,x)=-(x,x,y)$ we obtain

$\big(x,(x,x,y)\big)=0$ (16)

From $(x,J(x,y,x))=0$ we have

$(x,(y,x,x))=0$ (17)

From (2) and (17) we have

$((y,x),x,x)-0$ (18)

Interchanging x as y and y as x in above equation we have

$((x,y),y,y)=0$ (19)

Hence from equation (4), the identity (18) becomes

$(x,x,(y,x))=0$ (20)

From equation (18), (20) and (5), we get

$(x,(y,x),x)=0$ (21)

A ring satisfies the identity (5) also satisfy

$J(xy,x,y)=0$ (22)

By linearizing equation (1) gives

$Jx,,+Jy,x,z=0$

Hence $J(x,x,y)=0$ (23)

By linearizing this equation with respect to , we obtain from (5)

$J(xy,x,z)+J(xz,x,y)=0$

Put $z=x$ in above and by virtue of (2) we have

$J(x^2,x,y)=0$ (24)

And the identity we have $(x,yox,x)-xo(x,y,x)+yo(x,x,x)$ (25)

MAIN RESULTS:

LEMMA 1: If ring R of characteristic $\neq 2$ satisfies the identities (1), (8) and (22) and there are no nilpotent elements in R, then R satisfies the identity $(x,y,x) = 0$.

PROOF: Let us assume that $u = (x,y,x)$ and $(x,u,x) = (xu)x - x(ux)$.

From equation (19) we have $xu = ux$.

$$(x,u,x) = (xu)x - x(xu)$$

$$= [xu,x].$$

On the other hand, using equation (25), linearizing (8) and (1) we get

$$2[xu,x] = [xou,x] = [xo(x,y,x),x] = [(x,yox,x) - yo(x,x,x),x] = 0$$

There fore $(x,u,x) = (x,(x,y,x),x) = 0$ (26)

Substitute y^2 instead of y in (26) and using (8) and (25) we obtain,

$$0 = (x,(x,y^2,x),x) = (x,you,x) = yo(x,u,x) + uo(x,y,x) = uo(x,y,x)$$

By $(x,y,x) = u$, hence $u^2 = 0$ and $u = (x,y,x) = 0$. ♦

LEMMA 2: If the ring R is an antiflexible derivation alternator ring of characteristic$\neq$2 or 3 satisfies the identity (19) and (22), then R satisfies the identity

$$[x,y](x,x,y) = 0. \quad (27)$$

PROOF: We recall that the following identities in every ring

$$(x,[y,z],w) = ([x,y],z,w) + (x,y,[z,w]) - [x,(y,z,w)] - [(x,y,z),w] \quad (28)$$

$$J(x,y,z) = [xy,z] - x[y,z] - [x,z]y \quad (29)$$

$$2J(x,y,z) = [[x,y],z] + [[y,z],x] + [[z,x],y] \quad (30)$$

From (28) put $z = x$ and $w = y$ we obtain

$$(x,[y,x],y) = ([x,y],x,y) + (x,y,[x,y])$$

Adding $(x,[y,x],y)$ on both sides to equation and using flexible identity

$$2(x,[y,x],y) = ([x,y],x,y) + (x,y,[x,y]) + (y,[x,y],x)$$

$$= J([x,y],x,y) = 0 \text{ (from (22))}$$

Therefore $([x,y],x,y) = -(x,y,[x,y])$

$$= ([x,y],y,x) \text{ (by flexibility)}$$

On the other hand from linearized (19) we have

$$([x,y],y,x) + ([x,y],x,y) = 0$$

Comparing the identity with the previous one, we obtain

$$([x,y],x,y) = 0 \quad (31)$$

In (28) put $z = y$ and $w = x$, we obtain

$$([x,y],y,x) + (x,y,[y,x]) = [x,(y,y,x)] + [(x,y,y),x]$$

In view of (31) which gives

$[(x,y,y),x] = 0$ (32)

Finally setting $w = z = y$ in (28) we obtain

$(x,[y,y],y) = ([x,y],y,y) + (x,y,[y,y]) - [x,(y,y,y)] - [(x,y,y),y]$

From (19) which gives $[(x,y,y),y] = 0$ (33)

Further from $G(x,x,y)$ in view of (23) follows

$[x^2,y] = xo[x,y]$

Using (32) we get,

$[(x,y,y),x^2] = xo[(x,y,y),x] = 0$

Therefore $[(x,y,y),x^2] = 0$ (34)

By linearizing (33) we have

$[(x,x^2,y),y] + [(x,y,x^2),y] + [(x,y,y),x^2] = 0.$

In view of (34)

$[(x,x^2,y),y] + [(x,y,x^2),y] = 0.$

On the other hand linearizing (34) gives

$[(x,x^2,y),y] + [(x^2,x,y),y] = 0.$

From the last two equations we have

$[(x,x^2,y),y] = [(x^2,y,x),y] = [(y,x,x^2),y]$

Now using (24) we get

$3[(x,x^2,y),y] = [J(x,x^2,y),y] = 0$

Consequently

$[(x,x^2,y),y] = [(x^2,y,x),y] = [(y,x,x^2),y] = 0$ (35)

From (29) and (32) we have

$$[x,y](x,x,y) = [x(x,x,y),y] - x[(x,x,y),y] - J(x,(x,x,y),y)$$
$$= [x(x,x,y),y] - J(x,(x,x,y),y)$$

Using (9) we obtain

$x(x,x,y) = (x^2,x,y) + (x,x,xy) - (x,x2,y)$

Hence by virtue of (35)

$[x(x,x,y),y] = [(x,x,xy),y].$

Now linearizing (34) we obtain

$$[(x,x,xy),y] = -[(x,x,y),xy]$$
$$= -x[(x,x,y),y] - [(x,x,y),x]y + J(x,y,(x,x,y)) \text{ (from(29))}$$
$$= J(x,y,(x,x,y)) \text{ (from(34),(35))}$$

Therefore $[x,y](x,x,y) = 2J(x,y,(x,x,y))$

and $2J(x,y,(x,x,y)) = [[x,y],(x,x,y)] + [[y,(x,x,y)],x] + [[(x,x,y),x],y]$

(from (30))

$= [[x,y],(x,x,y)]$(from(34),(35))

$= -[y,(x,x,[x,y])] = 0.$

$[x,y](x,x,y) = 0.$

Lemma (2) is proved.♦

LEMMA 3: The ring R is an antiflexible derivation alternator ring satisfies identities

$(x,y)(x,x,z) + (x,z)(x,x,y) = 0$ (36)

$(w,y)(x,x,y) + (x,y)(w,x,y) + (x,y)(x,w,y) = 0$ (37)

and

$(w,y)(x,x,z) + (x,y)(w,x,z) + (x,y)(x,w,z) + (w,z)(x,x,y) + (x,z)(w,x,y) +$

$(x,z)(x,w,y) = 0.$ (38)

PROOF: By linearization (27) we obtain the equations (36) and (37).

The equation (38) can be obtained by linearization of either (36) or (37).♦

THEOREM 1: If the ring R is an antiflexible derivation alternator ring satisfies the identities (8), (5) and (18) is either commutative or alternative.

PROOF: From Lemma (1) that R is flexible, that is $(x,y,x) = 0.$

If R is not commutative then from Lemma (2)

$(x,x,y) = 0$ (39)

Which gives R is alternative.

Next we assume that R is not left alternative.

This implies that there exists elements x and y such that $(x,x,y) \neq 0$.

Again because of lemma (2) this means that

$(x,y) = 0.$ (40)

And also from (36) put $z = r$

$(x,y)(x,x,r) + (x,r)(x,x,y) = 0$

This implies $(x,r)(x,x,y) = 0$ (from (39))

Since (40) and using no-divisors of zero we obtain

$(x,R) = 0$ (41)

From (37) put $w = s$

$(s,y)(x,x,y) + (x,y)(s,x,y) + (x,y)(x,s,y) = 0.$

This implies $(s,y)(x,x,y) = 0.$

Since of (40) and using no-divisors of zero we obtain

$$(R,y) = 0 \qquad (42)$$

From (38) $z = r, w = s$ gives

$$(s,y)(x,x,r) + (x,y)(s,x,r) + (x,y)(x,s,r) + (s,r)(x,x,y) + (x,r)(s,x,y) + (x,r)(x,s,y) = 0.$$

Which implies $(s,r)(x,x,y) = 0$

By results of (41) and (42) and using no-divisors of zero once again we obtain

$$(R,R) = 0. \qquad (43)$$

from (43) implies that R is commutative.

Therefore if R is not alternative, then it must be commutative.♦

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