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Effects of Chemical Reaction and Aligned Magnetic Field on Unsteady MHD Casson Fluid Flow Past a Moving an Infinite Plate through a Porous Medium in The Presence of Thermal Radiation and Heat Absorption

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#### **ABSTRACT:**

The present paper is to study the effect of aligned magnetic field on an unsteady MHD free convection Casson fluid flow past an infinite plate through porous medium in the presence of thermal radiation and heat absorption. When  $t^*>0$ , the velocity  $u^*=u_0$ . At that time, the plate the temperature and the concentration are raised to  $T_w^*$  and  $C_w^*$ . A first order chemical reaction is taking into an account. A uniform magnetic field  $B_0$  is applied in  $y^*$ -direction. The systems of non-dimensional linear partial differential equations are solved by using the Laplace transform technique. The influences of various non-dimensional parameters on velocity, temperature, concentration, Sherwood numbers, Nusselt and skin friction coefficient have been discussed and analyzed through graphs and tables.

**KEYWORDS:** Thermal radiation, chemical reaction, Casson fluids, MHD, free convection and heat absorption.

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### **INTRODUCTION**

The study of non-Newtonian Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rates of shear, a yield stress below which no flows occurs and a zero viscosity at an infinite rate of shear. If a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, where as if a shear stress greater than yield stress is applied and it starts to move. Few examples of Casson fluids are jelly, tomato sauce, honey, concentrated fruit juice, blood etc. Casson model is sometimes stated to fit rheological data better than general viscoelastic model for many materials. Many authors have their research in Casson fluid for mathematical modeling. S. Ostrach 1 analyzed the laminar free-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force. Hall effects and chemical reaction on MHD convective flow through porous plate with variable suction & heat absorption was investigated by Masthanrao, S et al <sup>2</sup>. Pramanik, S <sup>3</sup> studied radiation effect of the Casson fluid and heat transfer flow past a porous stretching surface. J. L. McGregor <sup>4</sup> has done research on the Application of the Minimal Energy Hypothesis to a Casson Fluid. C. K. Kirubhashankar et al 5 have investigated Heat and mass transfer of chemical reaction an unsteady MHD flow of a Casson fluid in a parallel plate. Chemical reaction effect of mass transfer on MHD Flow of Casson fluid were studied by S. A. Shehzad et al <sup>6</sup>. Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer was analyzed by Das. M et al <sup>7</sup>. A. G. Vijaya kumar et al <sup>8</sup> were found the chemical reaction and radiation effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature and variable mass diffusion. Heat & mass transfer on unsteady MHD Casson fluid flow with convective boundary conditions was analyzed by K. Pushpalatha et al <sup>9</sup>. S.H.Reddy et al <sup>10</sup> were identified the radiation absorption and chemical reaction effect on MHD flow of heat generating Casson fluid past oscillating vertical porous plate. B.C.Sakiadas <sup>11</sup> investigated the boundary-Layer behavior on continuous solid surfaces. Diffusion of chemically reactive species in Casson fluid flow over an unsteady permeable stretching surface was analyzed by Vajravelu, K et al <sup>12</sup>. Free convection flow past an infinite plate were studied by Pop, I. and. Soundalgekar, V.M. 13. Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating was discussed by Hussanan A et al 14. Raptis et al 15 were found the unsteady free convective flow through a porous medium to infinite vertical plate using finite difference scheme. E.M.Arthur et al <sup>16</sup> have analyzed the Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field. R.K.Dash et al <sup>17</sup> were analyzed the shear augmented dispersion of a solute in a Casson fluid flowing in a conduit. M.G.Reddy 18 was studied an unsteady radioactive convective boundary layer flow of a Casson fluid with variable thermal conductivity. J.Prakash et al <sup>19</sup> were discussed on Dufour effects on unsteady hydro magnetic radiative fluid flow past a vertical plate through porous medium. C.S.K. Raju et al <sup>20</sup> have discussed the heat and mass transfer in MHD Casson fluid over permeable stretching surface. Chemical reaction and radiation effects on unsteady MHD free convection flow past an inclined moving plate with TGHS was investigated by S.Rama Mohan et al <sup>21</sup>. C. Veeresha et al <sup>22</sup> have analyzed the joul heating and diffusion effect on MHD radioactive and convective Casson fluid flow past an oscillating vertical porous plate.

In the present study, we analyzed the effect of radiation, chemical reaction and heat absorption on flow of Casson fluid past an infinite inclined plate through porous medium. The set of linear partial differential equations are solved analytically by using Laplace Transform technique. The influence of various non-dimensional quantities on the velocity, the temperature, the concentration, the skin friction, the local Nusselt and the Sherwood numbers are thoroughly investigated by graphs and tables.

### **MATHEMATICAL FORMULATION:**

Consider an unsteady MHD free convection heat & mass transfer flow of a viscous, incompressible, electrically, conducting, radiating and chemically reacting fluid past a semi infinite plate to the vertically. A uniform magnetic field of strength  $\mathsf{B}_0$  applied in a transverse direction to the fluid flow. Let x\*-axis is taken along the vertical plate and y\*-axis is taken normal to it. Initially, whent\*  $\leq 0$ , both the fluid and plate are at stationary condition having constant temperature and concentration. When  $t^*>0$ , the velocity  $u^*=u_0$ . At the same time, the plate temperature & concentration are raised to  $T_w^*$  and  $C_w^*$ . For free convection flow, it's also assumed that, the induced magnetic field is negligible as the magnetic Reynolds number of the flow is taken to be very small. The viscous dissipation is negligible in the energy equation. The effects of variation in density  $(\rho)$ with temperature & species concentration are considered only in the body force term, in accordance with usual Boussinesq approximation. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Since the flow of the fluid is assumed to be in the direction of x\*-axis, so the physical quantities are functions of the space co-ordinate y\*and t\*only. The rheological equation of state for an isotropic & incompressible flow of Casson fluid is as follows.

$$\tau_{ij} = \begin{cases} 2\left(\mu_{\alpha} + \frac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_{\alpha} + \frac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

here  $\pi = e_{ij}e_{ij}$  and  $e_{ij}$  are the  $(i,j)^{th}$  component of the deformation rate,  $\pi$  is the product of the component of the deformation rate with itself,  $\pi_c$  is a critical value of this product based on the non-Newtonian model,  $\pi_\alpha$  is plastic dynamic viscosity of non-Newtonian fluid, and  $P_y$  is the yield stress of the fluid.

Under the above assumptions the flow is governed by the following equations:

Momentum Equation:

$$\frac{\partial u^*}{\partial t^*} = v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u^*}{\partial y^{*2}} + g \beta_T^* (T^* - T_\infty^*) + g \beta_C^* (C^* - C_\infty^*) - \frac{\sigma B_0^2 \sin^2 \alpha}{\rho} u^* - \frac{v}{K} u^*$$
(1)

**Energy Equation:** 

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y^*} - \frac{Q^*}{\rho C_p} (T^* - T_{\infty}^*)$$
(2)

**Diffusion Equation:** 

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - Kr(C^* - C_{\infty}^*)$$
(3)

The initial and the boundary conditions are:

$$t^{*} \leq 0, u^{*} = 0, T^{*} = T_{\infty}^{*}, C^{*} = C_{\infty}^{*} \text{ for all } y^{*},$$

$$t^{*} > 0, u^{*} = u_{0}, T^{*} = T_{w}^{*} + (T_{w}^{*} - T_{\infty}^{*})e^{n^{*}t^{*}}, C^{*} = C_{w}^{*} + (C_{w}^{*} - C_{\infty}^{*})e^{n^{*}t^{*}} \text{ at } y^{*} = 0,$$

$$u^{*} \to 0, T^{*} \to T_{\infty}^{*}, C^{*} \to C_{\infty}^{*} \text{ as } y \to \infty.$$

$$(4)$$

Where  $u^*\beta_T^*$ ,  $\beta_C^*$ ,  $B_0$ , v,  $\kappa$ ,  $\rho$ ,  $T^*$ ,  $C^*$ ,  $C_p$ ,  $C_{s,r}$ ,  $q_r$ ,  $Q^*$ ,  $\sigma$ ,  $D_m$ , t, a,  $\beta$ , Sc, K, A, Pr and Kr are respectively the fluid velocity in the  $x^*$ - direction, coefficient of thermal expansion, coefficient of expansion with concentration, external magnetic field, kinematic viscosity, thermal conductivity, fluid density, temperature of the fluid, Species concentration, Specific heat at constant pressure, Concentration susceptibility, radioactive heat flux, heat absorption, electric conductivity, Coefficient

of mass diffusivity, time, aligned angle, Casson parameter, Schmidt number, porosity parameter, Suction parameter, Prandtl number and chemical reaction parameter.

The radioactive heat flux  $q_r$ , under Rosseland approximation of the form

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^*}{\partial y^*} \tag{5}$$

Here  $\sigma^*$  denotes the Stefan- Boltzmann constant & k\*denotes the mean absorption coefficient.

It is assumed that the temperature differences within the flow are sufficiently small and that  $T^{*^4}$  may be expressed as a linear function of the temperature. This is obtained by expanding  $T^{*^4}$  in a Taylor series about  $T^*_{\infty}$  and neglecting the higher order terms, thus we get

$$T^{*4} = 4T_{\infty}^{*3}T^* - 3T_{\infty}^{*4}$$
 (6)

From Eq's (5) and (6), Eq. (2) reduces to

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{16\sigma^* T_{\infty}^*}{3\rho C_p k^*} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q^*}{\rho C_p} (T^* - T_{\infty}^*)$$
(7)

On introducing the following non-dimensional quantities

$$u = \frac{u^{*}}{u_{0}}, y = \frac{u_{0}}{v} y^{*}, t = \frac{u_{0}^{2}}{v} t^{*}, K = \frac{k^{*} u_{0}^{2}}{v}, M = \frac{\sigma B_{0}^{2} v}{\rho u_{0}^{2}}, \theta = \frac{(T^{*} - T_{\infty}^{*})}{(T_{w}^{*} - T_{\infty}^{*})},$$

$$\phi = \frac{(C^{*} - C_{\infty}^{*})}{(C_{w}^{*} - C_{\infty}^{*})}, Gr = \frac{vg \beta_{r}^{*} (T_{w}^{*} - T_{\infty}^{*})}{u_{0}^{3}}, Gm = \frac{vg \beta_{c}^{*} (C_{w}^{*} - C_{\infty}^{*})}{u_{0}^{3}},$$

$$Pr = \frac{\rho v C_{p}}{\kappa}, K = \frac{k^{*} u_{0}^{2}}{v}, Q = \frac{Q^{*} v}{\rho C_{p} u_{0}^{2}}, R = \frac{16\sigma^{*} T_{\infty}^{*}}{3k^{*} k}, Sc = \frac{v}{D_{m}}, n = \frac{v}{u_{0}^{2}} n^{*}.$$

$$(8)$$

In view of (8) the Equations (1), (3) and (7), reduce to the following non-dimensional forms

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \left(M\sin^2\alpha + \frac{1}{K}\right) u + Gr\theta + Gm\phi \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+R}{\Pr}\right) \frac{\partial^2 \theta}{\partial y^2} - Q\theta \tag{10}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr \phi \tag{11}$$

The corresponding boundary conditions reduce to

$$t \le 0, \ u = 0, \theta = 0, \phi = 0 \text{ for all } y,$$
  
 $t > 0, u = 1, \theta = 1 + e^{nt}, \phi = 1 + e^{nt} \text{ at } y = 0,$   
 $u \to 0, \theta \to 0, \phi \to 0 \text{ as } y \to \infty$ 
(12)

#### **SOLUTION OF THE PROBLEM:**

The system of equations (9), (10) and (11) with subject to the boundary conditions in (12), are solved by using Laplace Transform technique and the expression for

$$\phi(y,t) = C_9 + C_{10} \tag{13}$$

$$\theta(y,t) = C_6 + C_7 \tag{14}$$

$$u(y,t) = a \left[ \frac{1}{b} + \frac{1}{b-n} \right] C_1 + c \left[ \frac{1}{d} + \frac{1}{d-n} \right] C_2 - \left[ \frac{a}{b} + \frac{c}{d} - 1 \right] C_3 - \left[ \frac{a}{b-n} + \frac{c}{d-n} \right] C_4$$
$$-a \left[ \frac{1}{b} + \frac{1}{b-n} \right] C_5 - \frac{a}{b} C_6 - \frac{a}{b-n} C_7 - c \left[ \frac{1}{d} + \frac{1}{d-n} \right] C_8 - \frac{c}{d} C_9 - \frac{c}{d-n} C_{10}$$
(15)

$$C_1 = \frac{e^{bt}}{2} \left[ \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} - \sqrt{(b+B_4)\,t}\right) + \exp\left(y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) \right] + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(b+B_4)}}{2\sqrt{t}} + \sqrt{(b+B_4)\,t}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_3(b+B_4)}\right) + \exp\left(-y\sqrt{B_4(b+B_4)}\right) + \exp\left(-y$$

$$C_2 = \frac{e^{dt}}{2} \left[ \exp\left(-y\sqrt{B_3(d+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} - \sqrt{(d+B_4)t}\right) + \exp\left(y\sqrt{B_3(d+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} + \sqrt{(d+B_4)t}\right) \right] + \exp\left(y\sqrt{B_3(d+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} + \sqrt{(d+B_4)t}\right) + \exp\left(y\sqrt{B_3(d+B_4)}\right) erfc\left(\frac{y\sqrt{B_4}}{2\sqrt{t}} + \sqrt{(d+B_4)t}\right) + \exp\left(y\sqrt{B_4}\right) erfc\left(\frac{y\sqrt{B_4}}{2\sqrt{t}} + \sqrt{(d+B_4)t}\right) +$$

$$C_{3} = \frac{1}{2} \left[ \exp\left(-y\sqrt{B_{3}B_{4}}\right) erfc\left(\frac{y\sqrt{B_{3}}}{2\sqrt{t}} - \sqrt{B_{4}t}\right) + \exp\left(y\sqrt{B_{3}B_{4}}\right) erfc\left(\frac{y\sqrt{B_{3}}}{2\sqrt{t}} + \sqrt{B_{4}t}\right) \right]$$

$$C_4 = \frac{e^{nt}}{2} \left[ \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} + \sqrt{(n+B_4)t}\right) \right] + \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(n+B_4)}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(n+B_4)}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_3(n+B_4)}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(-y\sqrt{B_3(n+B_4)}\right) + \exp\left(-y\sqrt{B_3(n+B_4)}\right) erfc\left(\frac{y\sqrt{B_4(n+B_4)}}{2\sqrt{t}} - \sqrt{(n+B_4)t}\right) + \exp\left(-y\sqrt{B_4(n+B_4)}\right) + \exp\left(-y\sqrt{B_4(n+B_$$

$$C_{6} = \frac{1}{2} \left[ \exp\left(-y\sqrt{B_{1}Q}\right) erfc\left(\frac{y\sqrt{B_{1}}}{2\sqrt{t}} - \sqrt{Qt}\right) + \exp\left(y\sqrt{B_{1}Q}\right) erfc\left(\frac{y\sqrt{B_{1}}}{2\sqrt{t}} + \sqrt{Qt}\right) \right]$$

$$C_7 = \frac{e^{nt}}{2} \left[ \exp\left(-y\sqrt{B_1(n+Q)}\right) erfc\left(\frac{y\sqrt{B_1}}{2\sqrt{t}} - \sqrt{(n+Q)t}\right) + \exp\left(y\sqrt{B_1(n+Q)}\right) erfc\left(\frac{y\sqrt{B_1}}{2\sqrt{t}} + \sqrt{(n+Q)t}\right) \right]$$

$$C_8 = \frac{e^{dt}}{2} \left[ \exp\left(-y\sqrt{Sc\left(d+kr\right)}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(d+Kr)t}\right) + \exp\left(y\sqrt{Sc(d+kr)}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(d+Kr)t}\right) \right] + \exp\left(y\sqrt{Sc(d+kr)}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(d+Kr)t}\right) + \exp\left(y\sqrt{Sc(d+kr)}\right) + \exp\left(y$$

$$C_9 = \frac{1}{2} \left[ \exp\left(-y\sqrt{Sc\,kr}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Kr\,t}\right) + \exp\left(y\sqrt{Sc\,kr}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Kr\,t}\right) \right]$$

$$C_{10} = \frac{e^{nt}}{2} \left[ \exp\left(-y\sqrt{Sc(n+kr)}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(n+Kr)t}\right) + \exp\left(y\sqrt{Sc(n+kr)}\right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(n+Kr)t}\right) \right]$$

Where

$$\begin{split} B_1 &= \frac{\Pr}{1+R}, B_2 = 1 + \frac{1}{\beta}, B_3 = \frac{1}{B_2}, B_4 = M + \frac{1}{K}, a = \frac{B_3 Gr}{B_1 - B_3}, b = \frac{B_3 B_4 - B_1 Q}{B_1 - B_3}, \\ c &= \frac{Gm \, B_3}{Sc - B_3}, d = \frac{B_3 B_4 - Sc \, Kr}{Sc - B_3}. \end{split}$$

#### **Sherwood Number:**

The rate of change of mass transfer is given by

$$Sh = -\left[\frac{\partial \phi}{\partial y}\right]_{y=0} \tag{16}$$

From equations (13) and (16), we get Sherwood number as follows

$$\phi(y,t) = D_0 + D_{10}$$

#### **Nusselt number:**

The rate of change of heat transfer is given by

$$Nu = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} \tag{17}$$

From equations (14) and (17), we get Nusselt number as follows

$$\theta(y,t) = D_6 + D_7$$

#### **Skin friction coefficient:**

The rate of change of velocity is given by

$$\tau = -\left(1 + \frac{1}{\beta}\right) \left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{18}$$

From equations (15) and (18), we get Skin friction as follows

$$\tau = -\left[1 + \frac{1}{\beta}\right] \begin{bmatrix} a\left[\frac{1}{b} + \frac{1}{b-n}\right]D_1 + c\left[\frac{1}{d} + \frac{1}{d-n}\right]D_2 - \left[\frac{a}{b} + \frac{c}{d} - 1\right]D_3 - \left[\frac{a}{b-n} + \frac{c}{d-n}\right]D_4 \\ -a\left[\frac{1}{b} + \frac{1}{b-n}\right]D_5 - \frac{a}{b}D_6 - \frac{a}{b-n}D_7 - c\left[\frac{1}{d} + \frac{1}{d-n}\right]D_8 - \frac{1}{d}D_9 - \frac{1}{d-n}D_{10} \end{bmatrix}$$

Here

$$D_{1} = e^{bt} \left[ \sqrt{B_{3}(b+B_{4})} erf\left(\sqrt{(b+B_{4})t}\right) + \sqrt{\frac{B_{3}}{\pi t}} e^{-(b+B_{4})t} \right]$$

$$D_{2} = e^{dt} \left[ \sqrt{B_{3}(d+B_{4})} erf\left(\sqrt{(d+B_{4})t}\right) + \sqrt{\frac{B_{3}}{\pi t}} e^{-(d+B_{4})t} \right]$$

$$D_3 = \sqrt{B_3 B_4} erf(\sqrt{B_4 t}) + \sqrt{\frac{B_3}{\pi t}} e^{-B_4 t}$$

$$D_{4} = e^{nt} \left[ \sqrt{B_{3}(n+B_{4})} erf\left(\sqrt{(n+B_{4})t}\right) + \sqrt{\frac{B_{3}}{\pi t}} e^{-(n+B_{4})t} \right]$$

$$D_{5} = e^{bt} \left[ \sqrt{B_{1}(b+Q)} erf\left(\sqrt{(b+Q)t}\right) + \sqrt{\frac{B_{1}}{\pi t}} e^{-(b+Q)t} \right]$$

$$D_6 = \sqrt{B_1 Q} \operatorname{erf} \left( \sqrt{Qt} \right) + \sqrt{\frac{B_1}{\pi t}} e^{-Qt}$$

$$D_{7} = e^{nt} \left[ \sqrt{B_{1}(n+Q)} \operatorname{erf}\left(\sqrt{(n+Q)t}\right) + \sqrt{\frac{B_{1}}{\pi t}} e^{-(n+Q)t} \right]$$

$$D_8 = e^{dt} \left[ \sqrt{Sc(d+Kr)} \operatorname{erf} \left( \sqrt{(d+Kr)t} \right) + \sqrt{\frac{Sc}{\pi t}} e^{-(d+Kr)t} \right]$$

$$D_9 = \sqrt{Sc \, Kr} erf\left(\sqrt{Krt}\right) + \sqrt{\frac{Sc}{\pi t}} e^{-Krt}$$

$$D_{10} = e^{nt} \left[ \sqrt{Sc(n+Kr)} \, erf\left(\sqrt{(n+Kr)t}\right) + \sqrt{\frac{Sc}{\pi t}} e^{-(n+Kr)t} \right]$$

Where

$$B_{1} = \frac{\Pr}{1+R}, B_{2} = 1 + \frac{1}{\beta}, B_{3} = \frac{1}{B_{2}}, B_{4} = M + \frac{1}{K}, a = \frac{B_{3}Gr}{B_{1} - B_{3}}, b = \frac{B_{3}B_{4} - B_{1}Q}{B_{1} - B_{3}},$$

$$c = \frac{GmB_{3}}{Sc - B_{3}}, d = \frac{B_{3}B_{4} - Sc\ Kr}{Sc - B_{3}}.$$

erfc = Complementary Error function

erf = Error function

#### **RESULT AND DISCUSSION:**

The systems of linear non-dimensional equations, with the boundary conditions are solved by using the Laplace Transform technique. The obtained results show the effects of the various non-dimensional governing parameters, such as Casson parameter (β), aligned angle (α) Magnetic parameter (M), Porosity parameter (K), Prandtl number (Pr), thermal Grashof number (Gr), mass Grash of number (Gm), thermal Radiation parameter (R), heat absorption parameter (Q), Schmidt number (Sc), chemical reaction parameter (Kr) and time (t) on the flow of velocity, temperature & concentration. Also Skin friction coefficient, Nusselt number and Sherwood number are presented in the tabular form. From **figures 1-18** for cooling (Gr>0, Gm>0) and heating (Gr<0,Gm<0) of the plate. The heating & cooling takes place by setting up free convection currents due to temperature and concentration gradient.

The influence of Magnetic parameter (M) and Prandtl number (Pr) on the fluid velocity distribution are shown **figures (1) and (2)** in the cases cooling and heating of the plate. It is observed that velocity decreases as M or Pr increases in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate. **Figure (3)** shows the effects of porosity parameter (K) on velocity distribution in case of cooling and heating of the plate. It is found that velocity increases as K increases in case of cooling of the plate. But a reverse effect is identified in case of heating of plate. From **figure (4)** reveals that the behavior of  $\alpha$  on velocity distribution in case of cooling and heating of the plate. It is clear that velocity increases as aligned angle ( $\alpha$ ) increases both cooling and heating of the plate. When  $\alpha = \frac{\pi}{2}$  we get velocity flow in y-direction. From **figure (5) and (6)** depicts the effects of thermal Grashof number (Gr), mass Grash of number (Gm) and heat absorption parameter (Q) on velocity distribution in cases cooling and heating of the plate. From this we observed that velocity increases as Gr or Gm or Q increases in case of cooling of the plate and a reverse effect is noticed in case of heating of the plate. The effect of thermal radiation (R) on velocity distribution is shown

**figure** (7) in cases cooling and heating of the plate. It is seen that velocity increase as R increases in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate. From **figures** (8)-(9) exhibits that the effects of Schmidt number (Sc) and chemical reaction (Kr) on velocity distribution in cases cooling and heating of the plate. It is found that velocity increases as Sc or Kr increases in case of cooling of the plate. But reverse effect is identified in case of heating of plate. The effects of Casson parameter ( $\beta$ ) on velocity distribution is shown **figure** (10) in case cooling and heating of the plate. It is observed that velocity increases initially and slowly downwards as  $\beta$  increases in case of cooling of the plate and it is noticed opposite phenomenon in case of heating of the plate. The influence of time (t) on velocity distribution are shown **figure** (11) in case cooling and heating of the plate. From this it is seen that velocity increases as time (t) increases in case of cooling and reverse phenomenon is observed in case of heating of the plate.

The effect of different flow parameters on the fluid temperature are given in **Figures** (12)-(15). The Prandtl number (Pr) is defined as the ratio of kinematic viscosity to thermal diffusivity. The influences of Prandtl number (Pr) and the heat source parameter (Q) on temperature distribution are shown **figures** (12)-(13). It is noticed that temperature decreases as Pr or Q increases on the fluid flow. From **figures** (14) shows that the effect of thermal radiation (R) on temperature. From this it is observed that temperature increases as R increases on the fluid flow. The influence of time (t) on temperature distribution in **figure** (15). It is clear that temperature increases as t increases on the fluid flow.

The influences of Schmidt number (Sc) and chemical reaction (Kr) on concentration distribution in **figures** (16) and (17). It is noticed that concentration distribution decreases as Sc or Kr increases on the fluid flow. From **figure** (18) depicts the effect of time (t) on concentration distribution. From this it is observed that concentration increases as t *increases* on the fluid flow.

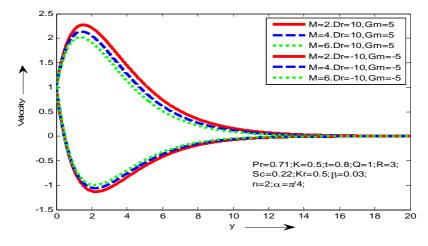


Fig.1.The effect of M on Velocity.

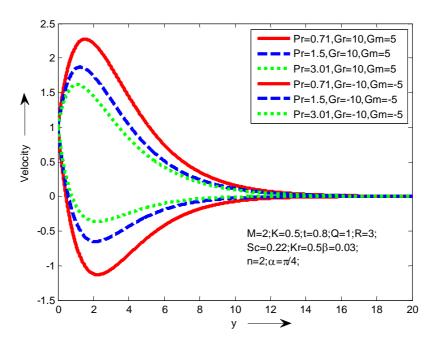


Fig.2.The effect of Pr on Velocity.

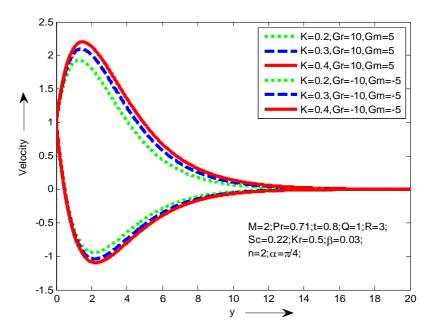


Fig.3.The effect of K on Velocity.

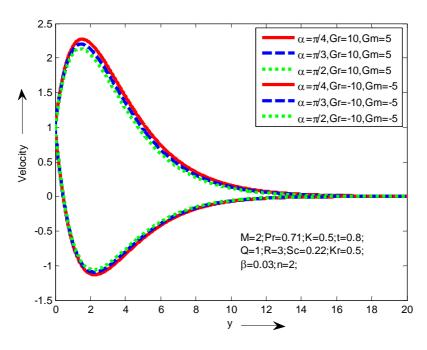


Fig.4.The effects of  $\alpha$  on Velocity.

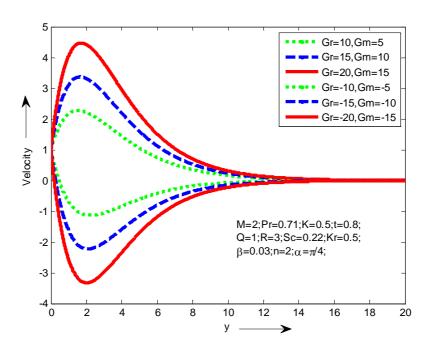


Fig.5.The effects of Gr and Gm on Velocity.

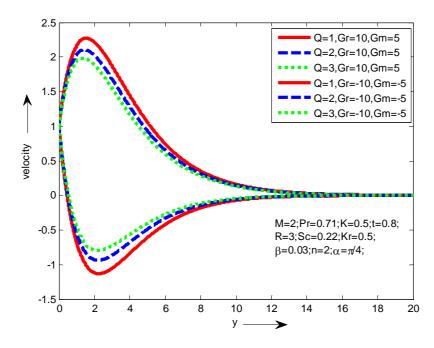


Fig.6.The effect of Q on Velocity.

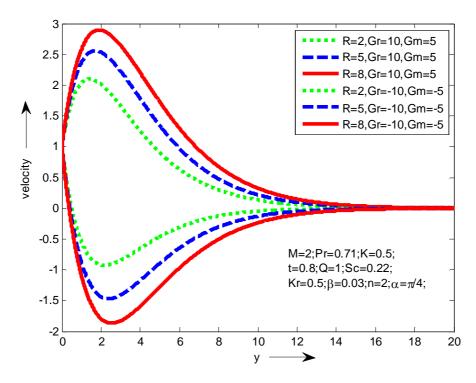


Fig.7.The effect of R on Velocity.

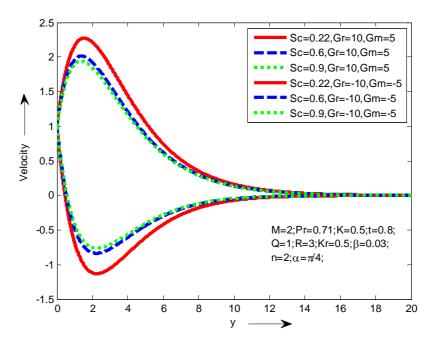


Fig.8.The effect t of Sc on Velocity.

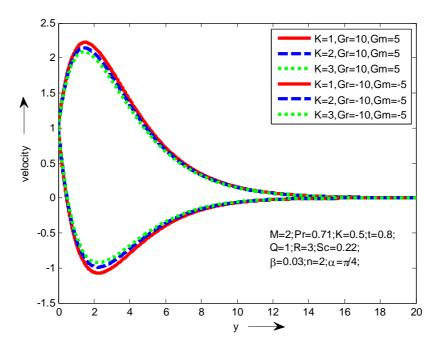


Fig.9.The effect of Kr on Velocity.

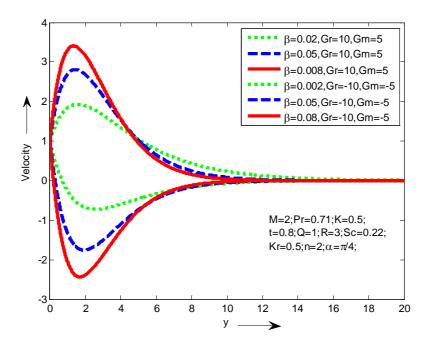


Fig.10.The effect of  $\beta$  on Velocity.

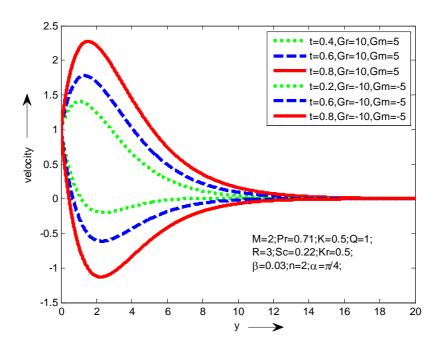


Fig.11. The effect of t on Velocity.

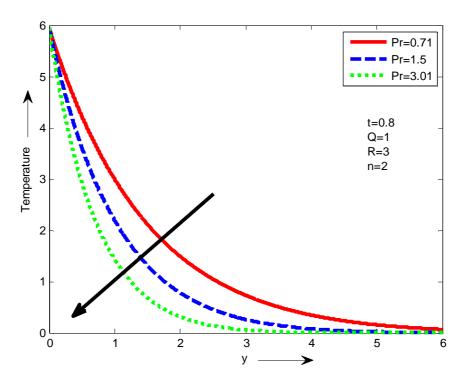


Fig.12.The effect of Pr on Temperature.

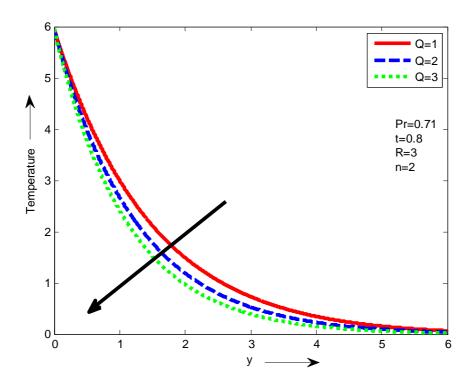
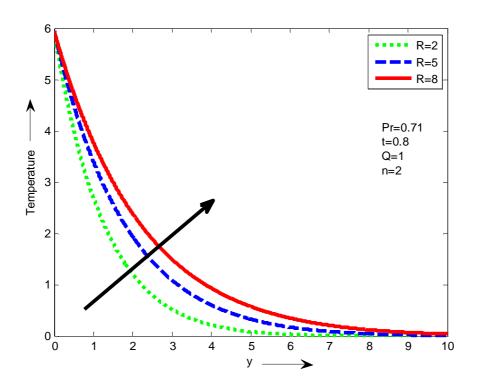


Fig.13.The effect of Q on Temperature.



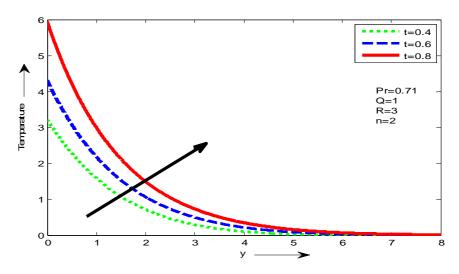


Fig. 14 The Effe ct of R on Temperature

Fig.15.The effect of t on Temperature.

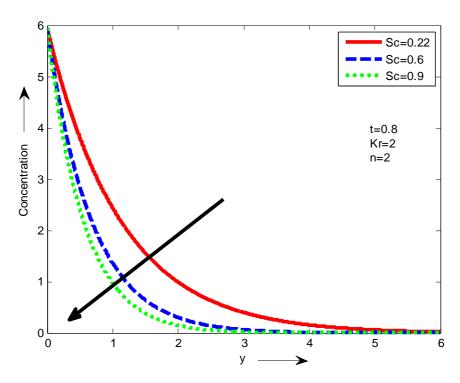


Fig.16.The effect of Sc on Concentration.

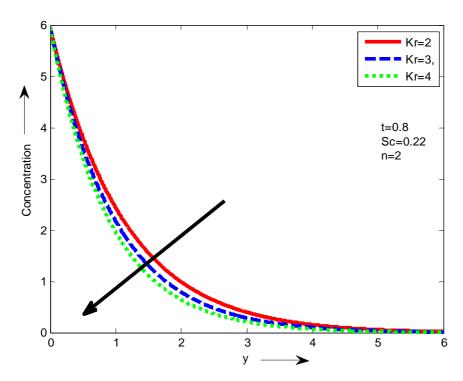


Fig.17.The effect of Kr on Concentration.

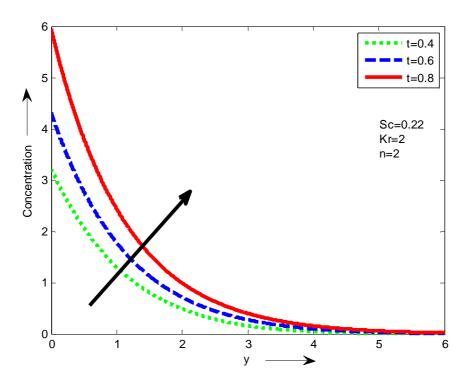


Fig.18.The effect of t on Concentration.

Table-1: Sherwood number

Sc	Kr	T	Sh
0.22	0.5	0.4	1.0740
0.6	0.5	0.4	1.7736
0.9	0.5	0.4	2.1722
0.22	0.3	0.4	1.0087
0.22	0.4	0.4	1.0415
0.22	0.5	0.6	0.9758
0.22	0.5	0.8	0.9206

Table-2: Nusselt number

Pr	R	Q	Т	Nu
0.71	3	1	0.4	1.1052
1.5	3	1	0.4	1.6064
3.01	3	1	0.4	2.2756
0.71	4	1	0.4	0.9885
0.71	5	1	0.4	0.9024
0.71	3	2	0.4	1.3628
0.71	3	3	0.4	1.5945
0.71	3	1	0.6	1.0436
0.71	3	1	0.8	1.0252

Effect of Casson parameter ( $\beta$ ), Schmidt number (Sc), magnetic parameter (M), thermal Grashof number (Gr), mass Grash of number (Gm), porosity parameter (K), Radiation parameter (R), chemical reaction parameter (Kr), heat absorption parameter (Q), Prandtl number (Pr), aligned angle ( $\alpha$ ) and time (t) on Skin friction coefficients, Nusselt number and Sherwood number is presented in tables.

**From table-1,** depicts the behavior of Sherwood numbers. Sherwood number increases as the values of Sc or Kr increases and it decreases as t increases.

**From table-2,** exhibits the influence of Nusselt numbers. Nusselt number increases as the values of Pr or Q increases and it reduces as the values R or t increases.

Table-3: Skin friction coefficient; (for cooling Gr>0,Gm>0, for heating Gr<0,Gm<0)

M	Pr	K	t	Q	R	Sc	Kr	β	α	τ (Gr=10, Gm=10)	τ (Gr= -10, Gm= -10)
2	0.71	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.5403	1.1514
3	0.71	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.5093	1.1628
4	0.71	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.4798	1.1736
2	1.5	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.4013	1.0125
2	3.01	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.2926	0.9037
2	0.71	0.3	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.4608	1.1806
2	0.71	0.4	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.5093	1.1628
2	0.71	0.5	0.6	1	3	2	0.4	0.03	$\pi/4$	-0.6835	1.2817
2	0.71	0.5	0.8	1	3	2	0.4	0.03	$\pi/4$	-0.7868	1.3807
2	0.71	0.5	0.4	2	3	2	0.4	0.03	$\pi/4$	-0.4951	1.1063
2	0.71	0.5	0.4	3	3	2	0.4	0.03	$\pi/4$	-0.4572	1.0684
2	0.71	0.5	0.4	1	4	2	0.4	0.03	$\pi/4$	-0.5856	1.1968
2	0.71	0.5	0.4	1	5	2	0.4	0.03	$\pi/4$	-0.6237	1.2349
2	0.71	0.5	0.4	1	3	3	0.4	0.03	$\pi/4$	-0.5000	1.1111
2	0.71	0.5	0.4	1	3	4	0.4	0.03	π/4	-0.4749	1.0861

2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.5375	1.1487
2	0.71	0.5	0.4	1	3	2	0.6	0.03	$\pi/4$	-0.5348	1.1460
2	0.71	0.5	0.4	1	3	2	0.4	0.04	$\pi/4$	-0.7220	1.4243
2	0.71	0.5	0.4	1	3	2	0.4	0.05	$\pi/4$	-0.8944	1.6759
2	0.71	0.5	0.4	1	3	2	0.4	0.03	π/6	-0.5728	1.1396
2	0.71	0.5	0.4	1	3	2	0.4	0.03	$\pi/3$	-0.5093	1.1628

The bold values in table indicate the variation of the parameters and variables present in the corresponding Colum's

From table-3: shows that the effects of Skin friction coefficient in case cooling and heating. From this it is clear that Skin friction coefficient increases as the values of Pr, Q, Sc, or Kr increases and it decreases as the values of R, t or  $\beta$  increases in case of cooling. But it is opposite phenomenon in case of heating. For both cases in case cooling and heating, Skin friction coefficient increases as the values of M or  $\alpha$  increases and Skin friction coefficients decreases as the values of K increases.

#### **Conclusion:** From the above work the following conclusions are made:

- \* Velocity increases with K, Gr, Gm, R or t increases in case of cooling and reverse phenomenon is observed in case of heating. Velocity decreases with M, Pr, Q, β, Sc or Kr increases in case of cooling and it is noticed opposite behavior in case of heating. But both the cases velocity increases with an aligned angle (α) increases.
- ❖ Temperature increases as R or t raises and it is reduces as Pr or Q rises.
- ❖ Concentration decreases as Kr or Sc increases and it is increases as time increases.
- ❖ Sherwood number increases as Sc or Kr increases and it decrease as t increases.
- Nusselt number increase as the values of Pr or Q, increases and it decay as R or t rises.
- \* Skin friction coefficient increases as the values of Pr, Q, Sc, or Kr increases and it is decreases as R, t or β increases. But the reverse effect is noticed in case of heating. For both cases in case cooling and heating, Skin friction coefficient increases as the values of M or α increases and Skin friction coefficient decreases as the values of K increases.

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