Research article **Available online www.ijsrr.org** **ISSN: 2279–0543**

International Journal of Scientific Research and Reviews

Multivalent Harmonic Function Associated With Salagean Operator

Noohi Khan (AP II)

Amity University Lucknow UP, India, Email: noohikhan2906@gmail.com

ABSTRACT

In this paper we define , a class HM(u,v a) of m-valent harmonic functions involving Salagean Operator[15] D_m^v is defined and studied. A subclass THM (u,v,a) of a class H(u,v a) is also been defined and studied. integral operator, convolution condition, for functions belonging to subclass THM(u,v,a) are obtained.

KEYWORDS : Multivalalent , Salagean , convolution , operator.

***Corresponding author**

Dr.Noohi Khan (AP II)

Amity University Lucknow UP

Email: noohikhan2906@gmail.com

1.INTRODUCTION

Definition 1.1

Let f be a harmonic function in a Jordan domain D with boundary C. Suppose f is continuous in $\bar{D}$ and $f(z) \neq 0$ on C. Suppose f has no singular zeros in D, and let m to be sum of the orders of the zeros of f in D. Then $\Delta_c \arg(f(z)) = 2\pi m$, where $\Delta_c \arg(f(z))$ denotes the change in argument of f(z) as z traverses C.

It is also shown that if f is sense-preserving harmonic function near a point z_0, where $f(z_0) = \omega_0$ and if $f(z) - \omega_0$ has a zero of order m $(m \geq 1)$ at z_0, then to each sufficiently small $\in > 0$ there corresponds a $\delta > 0$ with the property: "for each $\alpha \in N_\delta(\omega_0) = \{\omega : |\omega - \omega_0| < \delta\}$, the function $f(z) - \alpha$ has exactly m zeros, counted according to multiplicity, in $N_\in(z_0)$". In particular, f has the open mapping property that is, it carries open sets to open sets.

Let Δ be the open unit disc $\Delta = \{z : |z| < 1\}$ also let $a_k = b_k = 0$ for $0 \leq k < m$ and $a_m = 1$. Ahuja and Jahangiri [5, 9] introduce and studied certain subclasses of the family SH(m), $m \geq 1$ of all multivalent harmonic and orientation preserving functions in Δ. A function f in SH(m) can be expressed as $f = h + \bar{g}$, where h and g are of the form

$$1.1 \quad h(z) = z^m + \sum_{n=2}^{\infty} a_{n+m-1} z^{n+m-1}$$

$$g(z) = \sum_{n=1}^{\infty} b_{n+m-1} z^{n+m-1}, \quad |b_m| < 1.$$

According to above argument, functions in SH(m) are harmonic and sense-preserving in Δ if $J_f > 0 \text{ in } \Delta$. The class SH(1) of harmonic univalent functions was studied in details by Clunie and Sheil Small[16]. It was observed that m-valent mapping need not be orientation-preserving.

Let TH(m) denotes the subclass of SH(m) whose members are of the form

$$h(z) = z^m - \sum_{n=2}^{\infty} |a_{n+m-1}| z^{n+m-1}$$

and

$$g(z) = \sum_{n=1}^{\infty} |b_{n+m-1}| z^{n+m-1}, \quad |b_m| < 1.$$

Definition 1.2

For analytic function $h(z) \in S(m)$ Salagean [33] introduced an operator D_m^{ν} defined as follows:

$$D_m^0 h(z) = h(z), \; D_m^1 h(z) = D_m(h(z)) = \frac{z}{m} h'(z) \text{ and}$$

$$D_m^{\nu} h(z) = D_m(D_m^{\nu-1} h(z)) = \frac{z(D_m^{\nu-1} h(z))'}{m}$$

$$= z + \sum_{n=2}^{\infty} \left(\frac{n+m-1}{m}\right)^{\nu} a_{n+m-1} z^{n+m-1}, \; \nu \in N.$$

Whereas, Jahangiri et al. [17] defined the Salagean operator $D_m^{\nu} f(z)$ for multivalent harmonic function as follows:

(1.2) $$D_m^{\nu} f(z) = D_m^{\nu} h(z) + (-1)^{\nu} \overline{D_m^{\nu} g(z)}$$

where,

$$D_m^{\nu} h(z) = z^m + \sum_{n=2}^{\infty} \left(\frac{n+m-1}{m}\right)^{\nu} a_{n+m-1} z^{n+m-1}$$

$$D_m^{\nu} g(z) = \sum_{n=1}^{\infty} \left(\frac{n+m-1}{m}\right)^{\nu} b_{n+m-1} z^{n+m-1}.$$

Now, a sub class $H_m(\lambda,\nu,\alpha)$ of m-valent harmonic functions involving Salagean operator $D_m^{\nu}f(z)$ is defined as follows:

Definition 1.3

Let $f(z)=h(z)+\overline{g(z)}$ be the harmonic multivalent function of the form (1.1), then f belongs to HM(u,v,a) if and only if

$$(1.3)\quad \mathrm{Re}\left\{(1-\lambda)\frac{D_m^{\nu}f(z)}{z^m}+\lambda\frac{\frac{\partial}{\partial\theta}D_m^{\nu}f(z)}{\frac{\partial}{\partial\theta}z^m}\right\}>\alpha$$

where $0\le\alpha<1,\lambda\ge0$, $z=re^{i\theta}\in\Delta$ and $D_m^{\nu}f(z)$ is defined by (1.2) and

$$\frac{\partial}{\partial\theta}D_m^{\nu}f(z)=i\left[z(D_m^{\nu}h(z))'-(-1)^{\nu}\overline{z(D_m^{\nu}g(z))'}\right],\quad \frac{\partial}{\partial\theta}z^m=imz^m.$$

Denote the subclass THM(u,v,a) consist of harmonic functions $f_\nu=h+\overline{g}_\nu$ in HM(u,v,a) so that h and g_ν are of the form

$$(1.4)\quad h(z)=z^m-\sum_{n=2}^{\infty}|a_{n+m-1}|z^{n+m-1},$$

$$g_\nu(z)=(-1)^{\nu}\sum_{n=1}^{\infty}|b_{n+m-1}|z^{n+m-1},\ |b_m|<1.$$

Also note that THM(u,v,0)=THM(u,v).

The class HM(u,v,0) provides a transition between two classes:

$\mathrm{Re}\left\{\frac{D_m^{\nu}f(z)}{z^m}\right\}>\alpha$ and $\mathrm{Re}\left\{\frac{\frac{\partial}{\partial\theta}D_m^{\nu}f(z)}{\frac{\partial}{\partial\theta}z^m}\right\}>\alpha$ as λ moves between 0 and 1.

Denote HM(0,v,a) by PM(v,a) and HM(1,v,a) by QM(v,a) .

Definition 1.4

The generalized Bernardi-Libera-Livingston integral operator $L_c(f(z))$ for m-valent functions is defined by

$$L_c(f(z)) = \frac{c+m}{z^c}\int_0^z t^{c-1}h(t)dt + \overline{\frac{c+m}{z^c}\int_0^z t^{c-1}g(t)dt}, \qquad c > -1.$$

2. INTEGRAL OPERATOR

Let f belogs to THM(u,v,a) ; $\lambda \geq 1$. Thus $L_c(D_m^{\nu} f_{\nu}(z))$ belongs to the class THM(u,v,a).

Proof

From the representation of $L_c(f(z))$ it follows that

$$L_c(D_m^{\nu} f_{\nu}(z)) = \frac{c+m}{z^c}\int_0^z t^{c-1}D_m^{\nu}h(t)dt + \overline{\frac{c+m}{z^c}\int_0^z t^{c-1}\left(-1\right)^{\nu} D_m^{\nu}g_{\nu}(t)dt}$$

$$= \frac{c+m}{z^c}\int_0^z t^{c-1}\left(t^m - \sum_{n=2}^{\infty}|a_{n+m-1}|t^{n+m-1}\right)dt$$

$$+\overline{\frac{c+m}{z^c}(-1)^{\nu}\int_0^z t^{c-1}\left(\sum_{n=2}^{\infty}|b_{n+m-1}|t^{n+m-1}\right)dt}$$

$$= z^m - \sum_{n=2}^{\infty}A_{n+m-1}z^{n+m-1} + (-1)^{\nu}\sum_{n=1}^{\infty}B_{n+m-1}\overline{z}^{n+m-1}$$

where, $A_{n+m-1} = \frac{c+m}{n+m-1+c}|a_{n+m-1}|, B_{n+m-1} = \frac{c+m}{n+m-1+c}|b_{n+m-1}|$

Therefore,

$$\sum_{n=2}^{\infty}\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left\{\left(\frac{n+m-1}{m}\right)\lambda + (1-\lambda)\right\}\frac{c+m}{n+m-1+c}|a_{n+m-1}| + \right.$$

$$+\left\{\left(\frac{n+m-1}{m}\right)\lambda-(1-\lambda)\right\}\frac{c+m}{n+m-1+c}|b_{n+m-1}|\Bigg]$$

$$\leq\sum_{n=2}^{\infty}\left\{\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda+(1-\lambda)\right]|a_{n+m-1}|\right.$$

$$\left.+\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda-(1-\lambda)\right]|b_{n+m-1}|\right\}$$

$$\leq(1-\alpha)-(2\lambda-1)|b_m|$$

and so the proof is complete.

3.CONVOLUTION PROPERTY

Let fv belongs to THM(u,v,a) and Fv belongs to THM (u,v,a) ; $\lambda\geq 1$ then the convolution

$$(f_\nu * F_\nu)(z)=z^m-\sum_{n=2}^{\infty}|a_{n+m-1}A_{n+m-1}|z^{n+m-1}+$$
$$+(-1)^{\nu}\sum_{n=1}^{\infty}|b_{n+m-1}B_{n+m-1}|\bar{z}^{n+m-1}\in TH_m(\lambda,\nu,\alpha).$$

Proof

For Fv belongs to THM(u,v,a) so, $|A_{n+m-1}|\leq 1, |B_{n+m-1}|\leq 1$.

consider, $$\sum_{n=2}^{\infty}\left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda+(1-\lambda)\right]|a_{n+m-1}A_{n+m-1}|}{1-\alpha}\right]+$$

$$+\sum_{n=1}^{\infty}\left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda-(1-\lambda)\right]|b_{n+m-1}B_{n+m-1}|}{1-\alpha}\right]$$

$$\leq\sum_{n=2}^{\infty}\left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda+(1-\lambda)\right]|a_{n+m-1}|}{1-\alpha}\right]+$$

$$+\sum_{n=1}^{\infty}\left[\frac{\left(\frac{n+m-1}{m}\right)^{\nu}\left[\left(\frac{n+m-1}{m}\right)\lambda-(1-\lambda)\right]|b_{n+m-1}|}{1-\alpha}\right]$$

≤ 1 using equation coefficient inequality.

Therefore the result follows.

REFERENCENS

1. Ahuja OP. Jahangiri JM. Silverman H. Convolutions for special classes of Harmonic Univalent Functions. Appl. Math. Lett :2003;16(6): 905-909.
2. Ahuja OP. Silverman H. Extreme Points of families of Univalent Functions with fixed second coefficient. Colloq. Math: 1987;54:127-137.
3. Ahuja OP. Jahangiri JM. Harmonic Univalent Functions with fixed second coefficient .Hakkoido Mathematical Journal: 2002;31 :431-439.
4. Ahuja OP. Silverman H. Inequalities associating hypergeometric Functions with planar harmonic mappings: Mathematical journal of Analysis :2004;5(4):45-55.
5. Ahuja OP. Jahangiri. JM. Multivalent Harmonic star like Functions. Ann. Univ Mariae Curie – Sklodowska:2001; 55 (1):1-13.

6. Ahuja OP. Jahangiri JM. A subclass of Harmonic Univalent Functions. J. Natural geometry:2001;20:45-56.

7. Ahuja OP. Jahangiri. JM. Noshiro. Type Harmonic Univalent Functions. Math Japonica:2002;56(1): 1-7.

8. Ahuja, OP. Jahangiri, JM. Multivalent Harmonic convex Functions: submitted for publication.

9. Ahuja OP. Jahangiri, JM., Errata to "Multivalent Harmonic star like Functions. Ann. Univ. Mariae Curie-Sklodowska, Vol LV., 1 Sectio A 55 (2001), 1-3, Ann. Univ. Mariae Curie – Sklodowska:2002;56(1): 105-112.

10. AlA miri H. S. On a class of close to convex functions with negative coefficients. Mathematical, Tome :1989;31(45): 1-7.

11. AlA miri H. S. On P-close to convex functions of order α. Proc. Amer. Math. Soc:1971;29(3): 103-108.

12. Altintas O. Ertekin Y. A New subclass of analytic functions with negative coefficints. current topic in Analytic Function Theory World Scientific Publishing River Edge N.J.: 1992;53(3): 36-47.

13. Aouf M. K. Silverman H. Srivastava H.M. Some subclasses of Multivalent Functions involving a certain linear operator. Advanced studies in contemporary Mathematics: 2007; 14(2):215-232.

14. Chichra P N. New Subclasses of the class of close-to-convex Functions. Proc. Amer. Math. Soc.1977;62: 37-43.

15. Salagean GS. Subclass of Univalent Functions. Lecture Notes in Math. Springer – Verlag .1983;1013: 362-372.

16. Clunie J. Sheil–Small T. Harmonic Univalent Functions. Ann. Acad. Sci. Fenn. Ser. AI Math. 1984;9: 3-25.

17. Jahangiri JM Murugu sundar moorthy G. Vijaya K. Salagean type Harmonic Univalent Functions. South. J. Pure and Appl. Math. 2002;56(2):77-82.