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A Study on (1,2)*C And $(1,2)^* C^\#$ Sets

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ABSTRACT

The focus of this paper is to introduce a new class of sets namely $(1,2)^*$ C-closed set and $(1,2)^*$ $C^\#$- closed set in new bitopological setting. Also we investigate some of their properties.

KEYWORDS

$(1,2)^*$ bitopology, $(1,2)^*$ b-open, $(1,2)^*$ semi open, $(1,2)^*$ pre open, $(1,2)^*$ α-open $(1,2)^*$ β-open, $(1,2)^*$ regular open, $(1,2)^*$ semi regular, $(1,2)^*$ C- set, $(1,2)^*$ $C^\#$ - set, T_c space, $T_c^\#$ space.

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INTRODUCTION

The concept of a bitopological space (X, τ_1, τ_2) was first introduced by Kelly and the theory has been developed by different mathematician[8]. Their attention was mainly confined to the pairwise properties of the two topologies. When the research was going on towards pairwise properties in 1990 the endeavour of Lellis Thivagar brought a new idea on bitopological spaces[10]. In 2005 Lellis Thivagar and ravi introduced $(1,2)^*$ bitopological space[10]. The concept of $(1,2)^*$ b- open sets was introduced and studied by Sreeja and Janaki[11]. The purpose of this paper is to give a new type of open and closed sets namely, $(1,2)^*$ C set, $(1,2)^*$ $C^\#$ set. Also investigate some of its properties.

LITERATURE REVIEW

The bitopological space (X, τ_1, τ_2) was first introduced by Kelly and the theory has been developed by different mathematician[8]. Their attention was mainly confined to the pairwise properties of the two topologies. In 1990 the endeavour of Lellis Thivagar brought a new idea on bitopological spaces[10]. In 2005 Lellis Thivagar and ravi introduced $(1,2)^*$ bitopological space[10]. The concept of $(1,2)^*$ b- open sets was introduced and studied by Sreeja and Janaki[11]. The purpose of this paper is to give a new type of open and closed sets namely, $(1,2)^*$ C set, $(1,2)^*$ $C^\#$ set. Also investigate some of its properties.

PRELIMINARIES

Definition 1.2.1

Let $(X, \tau_{1,} \tau_2)$ be a bitopological space. A subset A of X is said to be $(1,2)^*$ b-open if $A \subseteq (\tau_{1,2}-int(\tau_{1,2}-cl(A))) \mathbf{U} (\tau_{1,2}-cl(\tau_{1,2}-int(A)))$. It is denoted by $(1,2)^*$ bo(X).

Definition 1.2.2

A subset S of a bitopological space $(X, \tau_{1,} \tau_2)$ is said to be $\tau_{1,2}$- open if S = A U B where $A \in \tau_1$ and $B \in \tau_2$.

Definition 1.2.3

A subset S of a bitopological space $(X, \tau_{1,} \tau_2)$ is said to be $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$- open

Definition 1.2.4

A subset A of a bitopological space $(X, \tau_{1,} \tau_2)$ is called

$(1,2)^*$ semi open if $A \subseteq \tau_{1,2}-cl(\tau_{1,2}-int(A))$

$(1,2)^*$ pre open if $A \subseteq \tau_{1,2}-int(\tau_{1,2}-cl(A))$

$(1,2)^*$ α-open if $A \subseteq \tau_{1,2} - int(\tau_{1,2} - cl(\tau_{1,2} - int(A)))$

$(1,2)^*$ β-open if $A \subseteq \tau_{1,2} - cl(\tau_{1,2} - int(\tau_{1,2} - cl(A)))$

$(1,2)^*$ regular open if $A = \tau_{1,2} - int(\tau_{1,2} - cl(A))$

$(1,2)^*$ semi regular if A is both $(1,2)^*$ semi open and $(1,2)^*$ semi closed.

$(1,2)^*$ generalized closed if $\tau_{1,2} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ – open in X.

$(1,2)^*$ semi generalized closed if $\tau_{1,2}$ –s cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ – open in X.

$(1,2)^*$ α generalized closed if $\tau_{1,2}$ –α cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ – open in X.

$(1,2)^*$ generalized α- closed if $\tau_{1,2}$ – α cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ –α- open in X.

$(1,2)^*$ generalized semi closed if $\tau_{1,2}$ – scl $(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition 1.2.6

A bitopological space (X, τ_1, τ_2) is called $(1,2)^*$ semi T_0 space if for any two distinct points x, y in X there exists a $(1,2)^*$ semi open set containing one but not the other.

$(1,2)^*$ T_b- space if every $(1,2)^*$ gs closed set is $\tau_{1,2}$- closed

$(1,2)^*$ α- space if every $(1,2)^*$ α- closed set is $\tau_{1,2}$- closed.

$(1,2)^*$ αT_b- space if every $(1,2)^*$ αg- closed set is $\tau_{1,2}$- closed.

MAIN WORK

$(1,2)^*$ C –Closed Sets And $(1,2)^*$ $C^\#$- Closed Sets

Definition 2.1

A subset A of a bitopological space (X, τ_1, τ_2) is called $(1,2)^*$ C-closed if $\tau_{1,2} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$ b-open in (X, τ_1, τ_2).

The complement of a $(1,2)^*$ C-closed set is called $(1,2)^*$ C-open.

Definition 2.2

A subset A of a bitopological space (X, τ_1, τ_2) is called $(1,2)^*$ $C^\#$- closed if $\tau_{1,2} - \alpha\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*$ C-open in (X, τ_1, τ_2).

The complement of a $(1,2)^*$ $C^\#$-closed set is called $(1,2)^*$ $C^\#$-open.

Theorem 2.3

(i) Every $\tau_{1,2}$ – closed set is $(1,2)^*$ C- closed.

(ii) Every $\tau_{1,2}$ – regular closed set is $(1,2)^*$ C- closed set.

(iii) Every $\tau_{1,2}$ – closed set is $(1,2)^*$ $C^{\#}$- closed

(iv) Every $(1,2)^*\alpha$-closed set is $(1,2)^*$ $C^{\#}$- closed.

(v) Every $(1,2)^*$ $C^{\#}$-closed set is $(1,2)^*$ αg-closed.

(vi) Every $(1,2)^*$ $C^{\#}$-closed set is $(1,2)^*$ gs-closed.

Proof

(i) Suppose U is $(1,2)^*$ b-open set such that $A \subseteq U$. Since A is $\tau_{1,2}$ – closed, $\tau_{1,2}$ – cl(A) $\subseteq$ U. Hence A is $(1,2)^*$ C- closed.

(ii) Suppose U is $(1,2)^*$ b-open set such that $A \subseteq U$. Since A is $\tau_{1,2}$ – regular closed, $\tau_{1,2}$ – Cl (int(A)) =A $\subseteq$ U. Hence A is $(1,2)^*$ C- closed.

(iii) Suppose U is $(1,2)^*$ C- open set such that $A \subseteq U$. Since A is $\tau_{1,2}$ – closed, $\tau_{1,2}$ – cl(A) =A $\subseteq$ U. We know that $\tau_{1,2}$ – α cl(A) $\subseteq$ $\tau_{1,2}$ – cl(A) $\subseteq$ U. Thus A is $(1,2)^*$ $C^{\#}$- closed.

(iv) Suppose U is $(1,2)^*$ C-open set such that $A \subseteq U$. Let A be $(1,2)^*\alpha$-closed set. Therefore $\tau_{1,2}$ – α cl(A) =A $\subseteq$ U. Hence A is $(1,2)^*$ $C^{\#}$- closed.

(v) Suppose U is $\tau_{1,2}$-open set such that $A \subseteq U$. Since A is $(1,2)^*$ $C^{\#}$-closed set, $\tau_{1,2}$ – αcl(A) $\subseteq$ U. We know that every $\tau_{1,2}$-open is $(1,2)^*$ C-open set. Hence A is $(1,2)^*\alpha$g-closed.

(vi) Suppose U is $(1,2)^*$ $\tau_{1,2}$-open set such that $A \subseteq U$. Let A be $(1,2)^*$ $C^{\#}$-closed set. Then $\tau_{1,2}$ – αcl (A) $\subseteq$ U .Since $\tau_{1,2}$ – scl(A) $\subseteq$ $\tau_{1,2}$ – α cl(A) $\subseteq$ U. Hence A is $(1,2)^*$ gs-closed.

Remark 2.4

However the converse of the above theorem need not be true may be seen by the following examples.

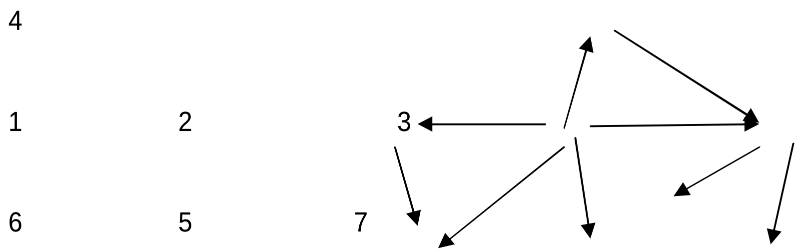
Example

X = { a, b, c }, τ_{1}= { ϕ, { a, b} , X}, τ_2 = { ϕ, { a,c }, X }, (1, 2)* $C^{\#}$- closed sets= { ϕ, {b},{c},{b, c}, X }. Here {b, c } is $(1,2)^*$ $C^{\#}$- closed set but not $(1,2)^*$ α- closed and $\tau_{1,2}$- closed. Because closure and alpha closure of {b,c} is not equal to {b,c}.

X = { a, b, c }, τ_{1}= { ϕ, { a} , X}, τ_2 = { ϕ, { b } , X }, (1, 2)* $C^{\#}$- closed sets= { ϕ, {c}, {a, c }, {b, c}, X }, (1, 2)* gs-closed sets= { ϕ, {a}, {b}, {c}, {a, c }, {b, c}, X }.Here {b} and {a } are $(1,2)^*$ gs-closed set but not $(1,2)^*$ $C^{\#}$- closed set.

The above results as shown by the following diagram

1. $(1,2)^*$ C- closed, **2.** $\tau_{1,2}$ – closed, 3. $(1,2)^*$ $C^\#$- closed, **4.** $(1,2)^*$ α-closed, 5. $(1,2)^*$ αg-closed, 6. $(1,2)^*$ gs-closed. 7. $\tau_{1,2}$ – regular closed.



Remark 2.5

The union and intersection of two $(1,2)^*$ $C^\#$- closed sets need not be $(1,2)^*$ $C^\#$- closed set as shown in the following example.

Example

X = { a, b, c }, τ_1= { ϕ, { a, b} , X}, τ_2 = { ϕ, {c}, {b,c}, {a,c} , X }, (1, 2)* $C^\#$- closed sets= { ϕ, {a},{b},{c}, {a,b }, X }..Here {b} and {c} are $(1,2)^*$ $C^\#$-closed set but {b, c} is not $(1,2)^*$ $C^\#$-closed set.

X = { a, b, c }, τ_1= { ϕ, { a} , X}, τ_2 = { ϕ, X }, (1, 2)* $C^\#$- closed sets= { ϕ,{b},{c}, {a,b},{b,c}, {a,c}, X }..Here {a,b} and {a,c} are $(1,2)^*$ $C^\#$-closed set but {a} is not $(1,2)^*$ $C^\#$- closed set.

Theorem 2.6

If a set A is $(1,2)^*$ $C^\#$-closed then $(1,2)^*$αcl(A)-A contains no nonempty $\tau_{1,2}$-closed set.

Proof

Let F be a $\tau_{1,2}$-closed subset of $(1,2)^*$αcl(A)-A. Therefore A ⊆ F^C and F ⊆ $(1,2)^*$αcl(A). F^C is $\tau_{1,2}$- open. Since every $\tau_{1,2}$- open set is $(1,2)^*$ C- open, F^C is $(1,2)^*$ C-open Let A be $(1,2)^*$ $C^\#$-closed. Then $(1,2)^*$αcl(A) ⊆ F^C whenever A ⊆ F^C. Thus F ⊆ $[(1,2)^*\alpha cl(A)]^C$. Thus F ⊆ [$(1,2)^*$αcl(A)] ∩ $[(1,2)^*\alpha cl(A)]^C$. Hence F = ϕ.

Theorem 2.7

If a set A is $(1,2)^*$ $C^\#$-closed then $(1,2)^*$αcl(A)-A contains no nonempty C-closed set.

Proof

Let F be a $(1,2)^*$closed subset of $(1,2)^*\alpha cl(A)$ - A. Therefore F ⊆ $\tau_{1,2}$ -αcl(A) – A and A ⊆ F^C and F^C is $(1,2)^*$C-open. Since A is $(1,2)^*$ $C^\#$-closed set, $(1,2)^*\alpha cl(A) \subseteq F^C$ whenever $A \subseteq F^C$. This implies that $F \subseteq [(1,2)^*\alpha cl(A)]^C$. Thus $F \subseteq [(1,2)^*\alpha cl(A)] \cap [(1,2)^*\alpha cl(A)]^C$. Hence F = ϕ.

Theorem 2.8

If A is a $(1,2)^*$ C-open and a $(1,2)^*$ $C^\#$-closed subset of (X, τ_1,τ_2) then A is a $(1,2)^*$ α-closed subset of (X, τ_1,τ_2).

Proof

Let A be $(1,2)^*$ C-open and a $(1,2)^*$ $C^\#$-closed subset of (X, τ_1,τ_2). Therefore $\tau_{1,2}$ - αcl(A) ⊆ A. We know that A ⊆ $\tau_{1,2}$ - αcl(A).This implies that $\tau_{1,2}$-αcl(A) =A. Hence A is a $(1,2)^*$ α-closed subset of (X, τ_1,τ_2).

Theorem 2.9

Let A be $(1,2)^*$ $C^\#$-closed subset of (X, τ_1,τ_2) if A ⊆ B ⊆ $(1,2)^*$ α-cl(A) then B is also a $(1,2)^*$ $C^\#$-closed subset of (X, τ_1,τ_2).

Proof

Suppose U is $(1,2)^*$ C–open such that B ⊆ U. Let A ⊆ B ⊆ U. Then A ⊆ U. Since A is $(1,2)^*$ $C^\#$ - closed set, $\tau_{1,2}$ - αcl(A) ⊆ U. But A ⊆ B ⊆ $(1,2)^*$ α-cl(A). Therefore $(1,2)^*$ α-cl(A) ⊆ $(1,2)^*$ α-cl(B). Hence $(1,2)^*$ α-cl(B) ⊆ U. Thus B is also a $(1,2)^*$ $C^\#$-closed subset of (X, τ_1,τ_2).

Theorem 2.10

For each a ε X either {a} is $(1,2)^*$ C-closed or $\{a\}^C$ is $(1,2)^*$ $C^\#$- closed.

Proof

Suppose {a} is not $(1,2)^*$ C-closed set in X Then $\{a\}^C$ is not $(1,2)^*$ C-open. Therefore the only $(1,2)^*$ C-open set containing $\{a\}^C$ is X and $(1,2)^*$ αcl $(\{a\}^C)$ ⊆ X. Hence $\{a\}^C$ is $(1,2)^*$ $C^\#$-closed set.

Theorem 2.11

Let A be $(1,2)^*$ $C^\#$- closed in X then A is $(1,2)^*\alpha$-closed if and only if $(1,2)^*$ αcl(A) – A is $\tau_{1,2}$-closed.

Proof

Suppose A is $(1,2)^*\alpha$-closed. Then A = $(1,2)^*\alpha$-cl(A). Therefore $(1,2)^*\alpha$-cl(A) –A = ϕ. Hence $(1,2)^*$ αcl(A) – A is $\tau_{1,2}$- closed.

Conversely, Suppose $(1,2)^* \alpha cl(A) - A$ is $\tau_{1,2}$- closed. Let A be $(1,2)^*$ $C^\#$- closed in X. By the Theorem 2.6 $(1,2)^*\alpha\text{-cl}(A) - A = \phi$. Then $(1,2)^*\alpha\text{-cl}(A) = A$. Hence A is $(1,2)^*\alpha$-closed.

Remark 2.12

For any subset A of a bitopological space (X, τ_1, τ_2) $(1,2)^*\alpha\text{-cl}(A^C) = [\,(1,2)^*\alpha\text{-int}(A)\,]^C$.

Theorem 2.13

A subset A of (X, τ_1, τ_2) is $(1,2)^*$ $C^\#$-open if and only if $F \subseteq (1,2)^*\alpha\text{-int}(A)$ whenever F is $(1,2)^*$C-closed and $F \subseteq A$.

Proof

Let $F \subseteq A$. Then $A^C \subseteq F^C$ and F^C is $(1,2)^*$ C-open. Since A^C is $(1,2)^*$ $C^\#$- closed, $(1,2)^*\alpha\text{-cl}(A^C) \subseteq F^C$. By using the Remark 2.12 $[\,(1,2)^*\alpha\text{-int}(A)\,]^C \subseteq F^C$. Hence $F \subseteq (1,2)^*\alpha\text{-int}(A)$.

Conversely, Let $A^C \subseteq U$ where U is $(1,2)^*$ C-open. Then $U^C \subseteq A$ where U^C is $(1,2)^*$ C-closed. By hypothesis $U^C \subseteq (1,2)^*\alpha\text{-int}(A)$. Therefore $[\,(1,2)^*\alpha\text{-int}(A)\,]^C \subseteq U$. By the Remark2.12 $(1,2)^*\alpha\text{-cl}(A^C) \subseteq U$. Hence A^C is $(1,2)^*$ $C^\#$-closed. Thus A is $(1,2)^*$ $C^\#$-open.

Theorem 2.14

If $(1,2)^*$ $\alpha\text{-int}(A) \subseteq B \subseteq A$ and A is $(1,2)^*$ $C^\#$-open then B is also $(1,2)^*$ $C^\#$-open.

Proof

Let $(1,2)^*$ $\alpha\text{-int}(A) \subseteq B \subseteq A$. This implies that $A^C \subseteq B^C \subseteq [\,(1,2)^*$ $\alpha\text{-int}(A)\,]^C$ .By the Remark 2.12 $A^C \subseteq B^C \subseteq (1,2)^*$ $\alpha\text{-cl}(A^C)$. Also A^C is $(1,2)^*C^\#$ -closed. By the Theorem 2.9 B^C is also $(1,2)^*$ $C^\#$ -closed. Hence B is $(1,2)^*$ $C^\#$ -open.

Remark 2.15

Every $\tau_{1,2}$- open set is $(1,2)^*$ $C^\#$-open. But the converse may not be true as shown in the following example.

Example

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a,b\}, X\}$, $\tau_2 = \{\phi, \{a,c\}, X\}$, $\tau_{1,2}$–open set = $\{\phi, \{a,b\}, \{a,c\}, X\}$, $(1,2)^*$ $C^\#$-open set $=\{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$. Here $\{a\}$ is $(1,2)^*$ $C^\#$-open set but not $(1,2)^*$ $\tau_{1,2}$–open.

Definition 2.16

A space (X, τ_1, τ_2) is called a $(1,2)^*$- $T_C^\#$ space if every $(1,2)^*$ $C^\#$ closed set in it is $(1,2)^*$ α-closed.

Theorem 2.17

Every $(1,2)^*$ $C^\#$-closed set is $(1,2)^*$ α-closed in $(1,2)^*$ T_1 space.

Proof

Let (X, τ_1,τ_2) be $(1,2)^*$ T_1 space and A be $(1,2)^*$ $C^\#$-closed set. Therefore for every x ε A there exists a $\tau_{1,2}$- open set U_x such that x ε U_x and y ∉ U_x . Then $\cup_{x\in A} Ux$ = U is $\tau_{1,2}$ – open. Therefore U is $(1,2)^*$ C – open also A ⊆ U and y ∉ U. Since A is $(1,2)^*$ $C^\#$-closed set, $\tau_{1,2}$ - αcl(A) ⊆ U whenever A ⊆ U. This implies that y ∉ $\tau_{1,2}$ - αcl(A). Then $\tau_{1,2}$ - αcl(A) ⊆ A. This implies that A = $\tau_{1,2}$ - αcl(A). Hence A is $(1,2)^*$ α-closed.

Theorem 2.18

For a space (X, τ_1, τ_2) the following condition are Equivalent.

(i) (X, τ_1, τ_2) is a $(1,2)^*$ $T_C^\#$ space.

(ii) Every singleton subset of (X, τ_1, τ_2) is either $(1,2)^*$ C-closed or $(1,2)^*$ α- open.

Proof

(i) →(ii) Let x ε X. Suppose {x} is not $(1,2)^*$ C-closed subset of (X, τ_1, τ_2). Then X – {x} is not a $(1,2)^*$ C-open set. So X is only $(1,2)^*$ C-open set containing X – {x}. So X-{x} is a $(1,2)^*$ $C^\#$-closed subset of (X, τ_1, τ_2). Let (X, τ_1, τ_2) be $(1,2)^*$ $T_C^\#$ space. Then X-{x} is a $(1,2)^*$ α-closed subset of (X, τ_1, τ_2). Hence {x} is a $(1,2)^*$ α- open subset of (X, τ_1, τ_2).

(ii)→(i) Let A be a $(1,2)^*$ $C^\#$ -closed set of X. Trivially A ⊆ $(1,2)^*$ αcl(A). Let x ε $(1,2)^*$ αcl(A). By (ii) {x} is either $(1,2)^*$ C-closed or $(1,2)^*$ α- open.

Case- A

{x} is $(1,2)^*$ C-closed. If x ∉ A, then $(1,2)^*$-αcl(A)-A contains a nonempty $(1,2)^*$C-closed set {x}. By theorem 2.7, we arrive at a contradiction. Thus x ε A.

Case – B

{x} is $(1,2)^*$-α open. Since x ε $(1,2)^*$-αcl(A), {x} ∩ A ≠ ϕ. This implies that x ε A. So $(1,2)^*$-αcl(A) ⊆ A. Therefore $(1,2)^*$-αcl(A) = A. Then A is $(1,2)^*$-α closed. Hence (X, τ_1, τ_2) is a $(1,2)^*$ $T_C^\#$ space.

Theorem 2.19

Every $(1,2)^*$ T_b- space is a $(1,2)^*$ $T_C^\#$ space.

Proof

Let A be a $(1,2)^*$ $C^\#$-closed set. Then by the Theorem 2.3, A is $(1,2)^*$ -gs-closed. Since (X, τ_1, τ_2) is a $(1,2)^*$- T_b- space, A is $\tau_{1,2}$- closed. It is true that every $\tau_{1,2}$- closed set is $(1,2)^*$-α-closed. Therefore X is a $(1,2)^*$ $T_C^\#$ space.

Remark 2.20

The converse of above theorem need not be true may be seen in the following example.

Example

$X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{ \phi, \{b\}, X \}$, $(1,2)^*$ $C^\#$-closed sets $=\{ \phi, \{c\}, \{a,c\}, \{b,c\}, X\}$ .Here all $(1,2)^*$ $C^\#$-closed sets are $(1,2)^*\alpha$-closed. Therefore X is $(1,2)^*$ $T_C{}^\#$ space. But it is not $(1,2)^*$ T_b space because $\{b\}$ is not $\tau_{1,2}$ –closed.

Theorem 2.21

Every $(1,2)^*$ αT_b- space is a $(1,2)^*$ $T_C{}^\#$ space.

Proof

Let A be a $(1,2)^*$ $C^\#$-closed set. Then by the Theorem 2.3, A is $(1,2)^*$ -αg-closed. Since (X, τ_1, τ_2) is a $(1,2)^*$- αT_b- space, A is $\tau_{1,2}$- closed. It is true that every $\tau_{1,2}$- closed set is $(1,2)^*$-α-closed. Therefore X is a $(1,2)^*$ $T_C{}^\#$ space.

Definition 2.22

A bitopological space (X, τ_1, τ_2) is called a $(1,2)^*$- T_C space if every $(1,2)^*$ C -closed set in it is $\tau_{1,2}$-closed.

Theorem 2.23

Let (X, τ_1, τ_2) be a bitopological space. If a set A is $(1,2)^*$ C- closed then $\tau_{1,2}$-cl(A) –A contains no non empty $(1,2)^*$ b-closed set.

Proof

Suppose $\tau_{1,2}$-cl(A) –A contains $(1,2)^*$ b-closed set F. Then $A \subseteq F^C$. F^C is $(1,2)^*$ b-open and A is $(1,2)^*$-C-closed. Therefore $\tau_{1,2}\text{-cl}(A) \subseteq F^C$. Then $F \subseteq [\tau_{1,2}\text{-cl}(A)]^C$. Hence $F \subseteq [\tau_{1,2}\text{-cl}(A)] \cap [\tau_{1,2}\text{-cl}(A)]^C = \phi$. This implies that $F = \phi$.

Theorem 2.24

For a bitopological space (X, τ_1, τ_2) the following condition are Equivalent.

(i) (X, τ_1, τ_2) is a $(1,2)^*$ T_C space.

(ii) Every singleton subset of (X, τ_1, τ_2) is either $(1,2)^*$ b-closed or $\tau_{1,2}$- open.

Proof

(i)→(ii) Let $x \varepsilon X$. Suppose $\{x\}$ is not $(1,2)^*$ b-closed subset of (X, τ_1, τ_2). Then $X – \{x\}$ is not a $(1,2)^*$ b-open set. So X is only $(1,2)^*$ b-open set containing $X – \{x\}$. So $X\text{-}\{x\}$ is a $(1,2)^*$ C-closed subset of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is a $(1,2)^*$ T_C space. Then $X\text{-}\{x\}$ is a $\tau_{1,2}$ -closed Hence $\{x\}$ is $\tau_{1,2}$- open.

(ii)→(i) Let A be a $(1,2)^*$ C -closed subset of X. Trivially A ⊆ $\tau_{1,2}$ -cl(A). Let x ε $\tau_{1,2}$ -cl(A). By (ii) {x} is either $(1,2)^*$ b-closed or $\tau_{1,2}$ - open.

Case - A

{x} is $(1,2)^*$ b-closed. If x ∉ A, then $\tau_{1,2}$ -cl(A) - A contains a nonempty $(1,2)^*$ b-closed set {x}. By the Theorem 2.23, we arrive at a contradiction. Thus x ε A.

Case - B

Suppose that {x} is $\tau_{1,2}$ -open. Since x ε $\tau_{1,2}$ -cl(A), {x} ∩ A ≠ ϕ. This implies that x ε A. So $\tau_{1,2}$-cl(A) ⊆ A. Therefore $\tau_{1,2}$ -cl(A) = A. Then A is $\tau_{1,2}$ -closed. Hence (X, τ_1, τ_2) is a $(1,2)^*$ T_C space.

Remark 2.25

$(1,2)^*$ $T_C^{\#}$ spaces and $(1,2)^*$ T_C space are independent of one another as the following example shows.

Example

X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = { ϕ, X }, $(1,2)^*$ C -closed sets ={ ϕ, {b,c}, X}, $(1,2)^*$ $C^{\#}$-closed sets ={ ϕ, {b}, {c}, {a,b}, {a,c}, {b,c}, X} . Here all $(1,2)^*$ C -closed sets are $\tau_{1,2}$ -closed. So (X, τ_1, τ_2) is a $(1,2)^*$ T_C space. But not $(1,2)^*$ $T_C^{\#}$ space. Because the set {a,c} is not $\tau_{1,2}$ -α closed.

CONCLUSION

In this study we discussed about two types of sets namely $(1,2)^*$ C-sed set and $(1,2)^*$ $C^{\#}$- set in new bitopological setting and two type of spaces, $(1,2)^*$ $T_C^{\#}$ spaces and $(1,2)^*$ T_C space are introduced. Also, some of their properties are investigated with some examples.

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