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Study Of Heat Transfer By Mhd Flow Near The Stagnation Point Over A Stretching Sheet In Fuzzy Environment

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ABSTRACT

A numerical investigation is done to study an MHD flow near the stagnation point over a stretching sheet with heat transfer. The governing partial differential equations with the boundary conditions are fuzzified using Zadeh's extension principle. The fuzzified governing equations along with the fuzzified boundary conditions are solved numerically using fourth order Runge-Kutta shooting method. The values of velocity and temperature for different involved parameters are tabulated.

KEYWORDS: MHD flow, stagnation point, fuzzy environment, triangular fuzzy number, extension principle.

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1. INTRODUCTION

The practical applications of the dynamics of fluid flow over a stretching surface are of utmost importance such as extrusion of plastic sheets, glass blowing, paper production, drying of papers and textiles, drawing plastic films, metal spinning, continuous casting and spinning of fibres etc. Since the quality of final products depends to a large extent on the skin friction coefficient and the surface heat transfer rate, so in all of the above cases, a study of the flow field and heat transfer can be significant importance. Many researchers have investigated various aspects of this problem, such as consideration of mass transfer, exponentially stretching surface, magnetic field and application to non-Newtonian fluids, and similarity solutions have been obtained.

Initially, Sakiadis[1] presented the boundary layer flow on a moving continuous solid surface. Later, Crane[2] studied a closed form solution of the two-dimensional flow over a stretching sheet by considering the stretching velocity proportional to the distance from the slot. The problems of the flow through stretching surface have been investigated by Wang[3], Troy *et al.*[4], Vajravelu and Nayfeh[5], Mukhopadhyay and Andersson[6] and Jat and Chaudhury [7] in various conditions. Makinde and Aziz [8], Mahapatra *et al.* [9] and Chaudhury *et al.* [10] analyzed the flow over stretching surface in different cases.

Stagnation point virtually appears in all flow fields of engineering and science, so stagnation point flow is a topic of significance in fluid mechanics. The stagnation region encounters the highest heat transfer, the highest pressure and highest rate of mass decomposition. Stagnation pont flow has various applications in many manufacturing processes in industry. The applications include the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, blood flow problems, processes in the textiles and the paper industries, flow over the tips of rockets, aircrafts, submarines and oil ships. The pioneering work in this area was carried out by Heimenz [11] who studied the steady boundary layer flow in the region of a stagnation point on an infinite wall. The extension to the axisymmetric case was presented by Homann[12]. Later, a large number of analytical and numerical studies explaining various physical situations of the boundary layer stagnation point flow are presented by Sparrow *et al*. [13], Chiam [14], Amin and Riley [15], Mahapatra and Gupta [16] and Wang [17]. Most recently, Rosali *et al*. [18], Mahapatra and Nandy [19] and Lok and Pop [20] considered the problem of stagnation point flow in various situations.

Flow through porous media has attracted a lot of attention because these are quite prevalent in nature. Such type of flow finds its applications in a broad spectrum of disciplines including chemical engineering and geophysics. It is also important in many technological processes, geothermal energy usage and in astrophysical problems. Many other applications may also benefit from a better understanding of fundamentals of mass, momentum, and energy transport in porous media, namely, petroleum reservoir operations, food processing, ooling of nuclear reactors, building insulation, underground disposal of nuclear waste, and casting and welding in manufacturing processes. Enhancement of forced convection by the use of a porous substrate has been the subject of several investigations. Comprehensive references on flow in porous media can be found in books by Ingham and Pop [21], Schlichting and Gersten [22], Vafai [23] and Nield and Bejan [24]. Moreover, Vafai and Kim [25] reported a composite system problem involving a relatively thin porous substrate attached to the surface of a flat plate.

In recent years, a number of simple fluid flow problems of viscous incompressible fluid have attained new attention in the more general context of magnetohydrodynamics. The desired properties of the end product and the rate of cooling can be controlled by the use of electrically conducting fluid and applications of magnetic field. The study of magnetohydrodynamic flow through a heated surface has important applications in many technological processes such as exotic lubricants and suspension solutions, magneto-hydrodynamic flight, foodstuff processing, MHD power generators, solidification of liquid crystals, the boundary layer control in aerodynamics, and in the field of planetary magnetosphere. Hydromagnetic boundary layer flow over a stretching surface has attracted attention of many researchers in recent time due to its important applications in metal-working processes and modern metallurgy. It seems that the magnetohydrodynamic flow over a stretching surface was first investigated by Andersson [26]. On the other hand, the problem of MHD stagnation point flow past a stretching sheet was presented by Mahapatra and Gupta [27]. Later, Abel and Mahesha [28], Ramesh *et al.*[29], Singh and Singh [30], Makinde *et al.* [31], Olajuwon and Oahimire [32], and Chaudhary and Kumar [33] analyzed and presented MHD flow problems considering various aspects of the problems.

In this article, an attempt has been made to fuzzify boundary value problems by taking an MHD flow problem near the stagnation point over a stretching sheet. We have observed the velocity and temperature distribution after fuzzifying the governing equations by Zadeh's extension principle. The fuzzified ordinary differential equations along with the fuzzified boundary conditions are solved by using fourth order Runge-Kutta shooting method by formulating suitable computer programs in MATLAB.

2. BASIC CONCEPT OF FUZZY SET THEORY

a. *Fuzzy Sets and Membership Function:*

Let X be a universal set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function denoted by $\mu_{\tilde{A}}$, i.e.,

$$\mu_{\tilde{A}}: X \to [0,1]$$

and$\mu_{\tilde{A}}$ is known as the membership grade of element x in fuzzy set $\tilde{A}$ for each $x \in X$.

If $X = \{x\}$ is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$$

where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singleton.

b. Fuzzy Number [16]:

A fuzzy number is a convex normalized fuzzy set defined on R whose membership function is piecewise continuous.

A triangular fuzzy number A can be defined as atriplet $[a, b, c]$. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x \le b \\ \dfrac{c-x}{c-b}, b \le x \le c \end{cases}$$

A trapezoidal fuzzy number A can be expressed as $[a, b, c, d]$and its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x \le b \\ 1, \quad\quad b \le x \le c \\ \dfrac{d-x}{d-c}, c \le x \le d \end{cases}$$

c. Zadeh's Extension Principle [34]:

A crisp function $f: X \to Y$ is said to be fuzzified when it is extended to act on fuzzy set defined on X and Y. i.e.,

$$\tilde{f}: \tilde{X} \to \tilde{Y}$$

and its inverse has the form $\tilde{f}^{-1}: \tilde{Y} \to \tilde{X}$.

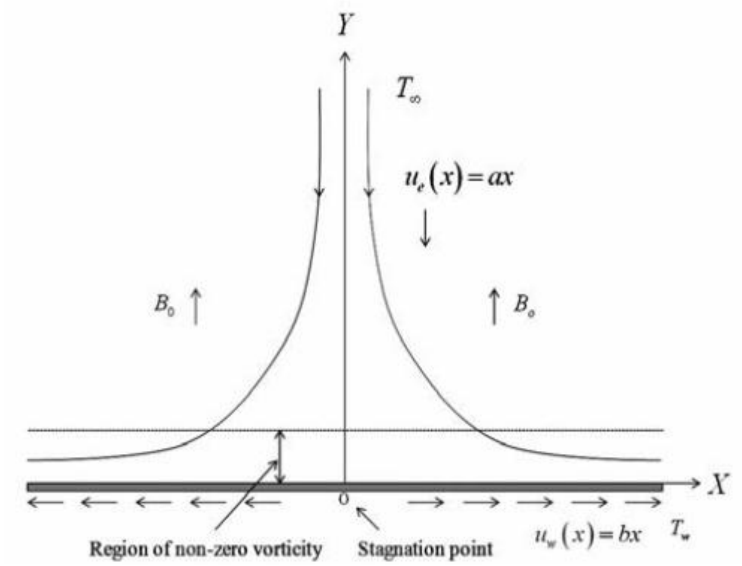
The extension principle states that for a given crisp function $f: X \to Y$ there induces two functions $\tilde{f}$ and $\tilde{f}^{-1}$ which are defined above for which membership functions are given by

$$\mu_{[\tilde{f}(A)](y)} = \sup_{x:y=f(x)} \mu_A(x), \text{ for all } A \in \tilde{X}$$

and $$\mu_{[\tilde{f}^{-1}(B)](x)} = \mu_B(\tilde{f}(x)), \text{ for all } B \in \tilde{Y}.$$

3. DESCRIPTION OF THE PROBLEM

We consider a steady, two-dimensional stagnation point flow of a viscous incompressible electrically conducting fluid impinging normally on a stretching surface of a constant temperature T_w in a porous medium. The stretching surface is placed along x-axis. The fluid is subjected to a uniform transverse magnetic field of strength B_0 in the direction of y-axis. The induced magnetic field is assumed to be negligible compared to the applied magnetic field. The external flow velocity varies linearly along x-axis, i.e., $u_e(x) = ax$, where $a > 0$ is the strength of the stagnation flow. The ambient fluid temperature T_∞ is constant. It is assumed that the velocity of the stretching surface is $u_w(x) = bx$, where $b > 0$ is the stretching rate.



Fig, 1: Flow configuration

With the above assumptions the governing boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\nu}{K_1}(u_e - u) + \frac{\sigma B_0^2}{\rho}(u_e - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho C_p}(u_e - u)^2 \tag{3}$$

Boundary conditions are-

$$\left.\begin{array}{l} y = 0: u = u_e(x) = bx, T = T_w \\ y \to \infty: u = u_e(x) = ax, T = T_\infty \end{array}\right\} \tag{4}$$

where u and v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, K_1 is the permeability of the porous medium, σ is the electrical conductivity, ρ is the fluid density, T is the temperature of the fluid, α is the thermal diffusivity, μ is the coefficient of viscosity and C_p is the specific heat at constant pressure.

To transform the above governing equations into ordinary differential equations, following similarity transformations are introduced.

$$\psi(x, y) = \sqrt{\alpha x u_e} f(\eta) \tag{5}$$

$$\eta = \sqrt{\frac{u_e}{\alpha x}} y \tag{6}$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta) \tag{7}$$

where $\psi(x,y)$ is the stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ which automatically satisfy the equation (1), $f(\eta)$ is the dimensionless stream function, η is the similarity variable and $\theta(\eta)$ is the dimensionless temperature.

Using (5)-(7) in equation (2) and (3) we have

$$Prf''' + ff'' - f'^2 + (1 - f')(K + M) + 1 = 0 \tag{8}$$

$$\theta'' + f\theta' + Ec\{Prf''^2 + M(1 - f')^2\} = 0 \tag{9}$$

Corresponding boundary conditions are-

$$\left.\begin{array}{l} \eta = 0: f = 0: f' = c, \theta = 1 \\ \eta \to \infty: f' = 1, \theta = 0 \end{array}\right\} \tag{10}$$

where primes denote differentiation with respect to η, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $K = \frac{\nu}{aK_1}$ is the permeability parameter, $M = \frac{\sigma B_0^2 \nu Re_x}{\rho u_e^2}$ is the magnetic field parameter, $Re_x = \frac{u_e x}{\nu}$ is the local Reynolds number, $Ec = \frac{u_e^2}{C_p(T_w - T_\infty)}$ is the Eckert number and $c = \frac{b}{a}$ is the stretching parameter with $c > 0$.

4. FUZZIFICATION OF THE PROBLEM

Every physical problem is inherently biased by uncertainty. There is often a need to model, solve and interpret the problems one encounters in the world of uncertainty. In general, science and engineering systems are modelled to ordinary and partial differential equations, but the type of differential equation depends upon the application, domain, complicated environment, the effect of coupling and so on. In recent years, this subject has become an important area of research due to its wide range of applications in various disciplines, namely physics, chemistry, applied mathematics, biology, economics, and in engineering systems such as fluid mechanics, viscoelasticity, civil, mechanical, aerospace, chemical etc. [35].

In general, parameters, variables and initial conditions involved in the model are considered as crisp or defined exactly for easy computation. However, rather than the particular value, we may have only the vague, imprecise and incomplete information about the variables and parameters being a result of errors in measurement, observations, experiment, applying different operating conditions, or it may be maintenance-induced errors, which are uncertain in nature. So, to overcome these uncertainties and vagueness, one may use interval and fuzzy set theory. Interval and fuzzy set theory refers to the uncertainty when we may have lack of knowledge or incomplete information about the variables and parameters.

Several authors have used the concept of fuzzy set theory in the field of differential equations.Fuzzy differential equations are used in modelling problems in science and engineering. It represents a proper way to model dynamical systems under uncertainty and vagueness. Kaleva [36] first gave the idea of fuzzy differential equations. His paper dealt with fuzzy set-valued mappings of a real variable whose values are normal, convex, upper semi-continuous and compactly supported fuzzy sets in. Gomes *et al.* [37] discussed fuzzy differential equations in various approaches. Chakraverty *et al.* [35] and Kermani and Saburi [38] have discussed some numerical methods for fuzzy fractional differential equation and fuzzy partial differential equation respectively. Chalco-Cano *et al.*[39] also studied about the solution of fuzzy differential equations.

We have used Zadeh's extension principle to extend the above ordinary differential equation into fuzzy form as below:

$$\widetilde{Pr}\tilde{f}''' + \tilde{f}\tilde{f}'' - \tilde{f}'^2 + (1 - \tilde{f}')(\tilde{K} + \tilde{M}) + 1 = 0 \qquad (11)$$

$$\tilde{\theta}'' + \tilde{f}\tilde{\theta}' + \widetilde{Ec}\left\{\widetilde{Pr}\tilde{f}''^2 + \tilde{M}(1 - \tilde{f}')^2\right\} = 0 \qquad (12)$$

Boundary conditions are-

$$\left.\begin{array}{l}\eta = 0: \tilde{f} = 0, \tilde{f}' = \tilde{c}, \tilde{\theta} = 1\\ \eta \to \infty: \tilde{f}' = 1, \tilde{\theta} = 0\end{array}\right\} \qquad (10)$$

5. RESULTS AND DISCUSSION

The system of fuzzified ordinary differential equations (12) and (13) together with the boundary conditions (14) is solved numerically by fourth order Runge-Kutta shooting method. We have considered the triangular fuzzy number in solving the equations. The values of the various parameters involved in the problem are taken as $\tilde{M} = 0.5, \widetilde{Pr} = 0.71, \tilde{K} = 0.05, \widetilde{Ec} = 0.3$ and $\tilde{c} = 0.5$ which are considered in the computer program as $\tilde{M} = [0.5,0.5,0.5], \widetilde{Pr} = [0.71,0.71,0.71], \tilde{K} = [0.05,0.05,0.05], \widetilde{Ec} = [0.3,0.3,0.3]$ and $\tilde{c} = [0.5,0.5,0.5]$.

The following tables give the values of velocity and temperature for increasing values of η.

Table I: Values of velocity distribution for different values of η

η	Taking different initial value			Taking same initial value		
	Left Value	Mid Value	Right Value	Left Value	Mid Value	Right Value
0.0	-0.000100	0.000000	0.000100	0.000000	0.000000	0.000000
0.1	0.158695	0.159802	0.160796	0.159000	0.158988	0.159001
0.2	0.287996	0.289130	0.290099	0.288515	0.288471	0.288519
0.3	0.392837	0.394025	0.396945	0.393588	0.393494	0.393598
0.4	0.477561	0.478838	0.479677	0.478575	0.478413	0.478591
0.5	0.545896	0.547306	0.549024	0.547214	0.546967	0.547239
0.6	0.601020	0.602617	0.603165	0.602699	0.602348	0.602734
0.7	0.645636	0.647483	0.649804	0.647747	0.647271	0.647794
0.8	0.682036	0.684209	0.686233	0.684668	0.684041	0.684730
0.9	0.712160	0.714749	0.716395	0.715424	0.714618	0.715505
1.0	0.737658	0.740770	0.744941	0.741690	0.740667	0.741793
1.1	0.759938	0.763699	0.766280	0.764901	0.763617	0.765029
1.2	0.780214	0.784769	0.788628	0.786301	0.784705	0.786460
1.3	0.799543	0.805066	0.809045	0.806984	0.805015	0.807182
1.4	0.818868	0.825558	0.829477	0.827935	0.825518	0.828177
1.5	0.839045	0.847135	0.855780	0.850054	0.847104	0.850349
1.6	0.860870	0.870632	0.880758	0.874191	0.870607	0.874550
1.7	0.885110	0.896856	0.910178	0.901173	0.896837	0.901608
1.8	0.912520	0.926609	0.937801	0.931821	0.926596	0.932344
1.9	0.943862	0.960707	0.977394	0.966972	0.960700	0.967600
2.0	0.979924	1.000000	1.211750	1.007499	1.000000	1.008250

Table II: Values of temperature distribution for different values of η

η	Taking different initial value			Taking same initial value		
	Left Value	Mid Value	Right Value	Left Value	Mid Value	Right Value
0.0	0.990000	1.000000	1.001000	1.000000	1.000000	1.000000
0.1	0.928304	0.939308	0.948304	0.938392	0.938393	0.938396
0.2	0.865393	0.876410	0.887394	0.875567	0. 875569	0.875581
0.3	0.801811	0.812850	0.823814	0.812071	0. 812074	0.812102
0.4	0.738049	0.749120	0.758056	0.748394	0. 748399	0.748450
0.5	0.674568	0.685681	0.696578	0.684997	0. 685005	0.685085
0.6	0.611808	0.622973	0.633823	0.622321	0. 622333	0.622447
0.7	0.550193	0.561420	0.572213	0.560791	0. 560807	0.560961
0.8	0.490128	0.501428	0.511155	0.500813	0.500833	0.501031
0.9	0.431993	0.443377	0.454028	0.442768	0. 442793	0.443038
1.0	0.376140	0.387618	0.398183	0.387007	0.387036	0.387331
1.1	0.322878	0.334460	0.346931	0.333841	0.333876	0.334222
1.2	0.272475	0.284172	0.296539	0.283540	0.283580	0.283977
1.3	0.225149	0.236969	0.247224	0.236322	0.236367	0.236814
1.4	0.181065	0.193018	0.205152	0.192353	0.192403	0.192897
1.5	0.140333	0.152427	0.164432	0.146744	0.147798	0.148335
1.6	0.103008	0.0115252	0.120122	0.114551	0.105609	0.112185
1.7	0.069095	0.081493	0.093222	0.080837	0.060837	0.079448
1.8	0.038544	0.051103	0.065686	0.050538	0.030436	0.040076
1.9	0.011264	0.023984	0.034421	0.020343	0.024309	0.031972
2.0	-0.012882	0.000000	0.012710	-0.000749	0.000000	0. 000981

6. CONCLUSION

The initial right value, left value and the mid values are taken from the given boundary conditions. As the velocity and temperature progresses from the initial point computations give the left and the right values which are respectively to the left and the right of the mid value as desired.

So, from the above observations, we can conclude that the mid value of a triangular fuzzy number coincides with the crisp value of the original problem.

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