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Stochastic Processes and their Applications in Music: A Review

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ABSTRACT

This paper presents a review of the connections between music and statistics with particular reference to the application of stochastic processes in music composition. It explores some studies on how stochastic principles can be used to generate music and analyze music pieces. The term Stochastic Music was first coined by Iannis Xenakis in 1956 and he is now recognized as a pioneer of introducing probabilistic elements and their scope in music composition. In his book 'Formalized Music: thought and Mathematics in Composition', he gave a detailed analysis of generation of music using stochastic processes, specially Markov Chains, thus laying the foundations of the idea that music composition can be a purely stochastic process. This study briefly traces the history of the development of stochastic music. The paper also presents a discussion on some applications of the basic principles of stochastic processes based on a review of literary studies in this area focusing on examples of how these techniques have been implemented to classify, create and identify musical compositions.

KEYWORDS: Music, composition, stochastic, Markov Chain.

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1. INTRODUCTION

Harmonies between music and mathematics existed since time immemorial. The ancient Greeks made significant contributions to music theory and gave equal importance to the study of music along with mathematics. In particular, Pythagorus, the ancient Greek mathematician and philosopher made significant contributions to the development of not only mathematics but also to music theory and is recognized as the first music theorist.He discovered the diatonic scale the eight-note musical scale popularly recognized as the basic notes of music, "Do–Re–Mi–Fa–So–La–Ti–Do". In doing so, he established the relation between music intervals and pairs of integers. Along with Pythagorus, Plato and Aristotle were influential figures when establishing the historical connections between music and mathematics. It was their schools of thought that considered music as an integral part of the study of mathematics. The Pythagorean curriculum known as the Quadrivium (figure 1) clearly places music together at the same level as the three other components arithmetic, geometry and astronomy[1].According to Proclus, 'Arithmetic studies quantities, music the relation between the quantities'.

Figure 1:

Quadrivium[1]

Mathematics (the study of the unchangeable)			
Quantity		Magnitude	
Alone (the absolute) Arithmetic	In relation (the relative) Music	At rest (the stable) Geometry	In motion (the moving) Astronomy

Much later in the eighteenth century, the great French Musicologist Jean-Philippe Rameau, established a synthesis between music as a creative art and music as a mathematical science. Rameau proved that the entire sequence of positive numbers is contained in music in the form of harmonies.

Mathematical language and notions have shaped the language and many concepts of music theory. Music makes use of a symbolic language, together with a rich system of notation, including diagrams which are similar to mathematical graphs of discrete functions in two-dimensional Cartesian coordinates (the x-coordinate representing time and the y-coordinate representing pitch) and Music theorists used these "cartesian" diagrams long before they were introduced in geometry[2].Mathematical patterns can be found in musical compositions and the connection between them has been established in many studies. Examples are the Fibonacci sequence and fractals. There are instances of Fibonnaci sequence being used in music. Fractal geometry has received considerable attention from composers and researchers to generate music composition. Fractals are geometrical patterns that are similar to themselves. This self-similarity can be exact, approximate or statistical, that repeats a pattern stochastically. Fractals have been known to exist in images, landscapes and sound. Ice crystals, flowers, fruits like pineapples, pine cones are all known to grow according to fractal laws. Fractal music methods can be used to produce a wide variety of songs. In particular the 1/f distribution and its application in creating music compositions, first suggested by Mandelbrot in his famous treatise The Fractal Geometry, has been the subject of extensive research [3, 4]. A recent study by Levitin et. al. [5] has produced some interesting results. The researchers analyzed close to 2000 musical movements from 558 compositions spanning four centuries and dozens of composers and styles. The study concluded that all music compositions they studied shared the same fractal quality i.e. four centuries of musical compositions obey the same mathematical rule that are found throughout the natural world.

2. DEVELOPMENT OF ALGORITHMIC MUSIC

The initial ideas of algorithmic music were developed from the concept of aleatoric music. Aleatoric music is defined as music employing the elements of chance in the choice of tones, rests, durations, rhythms, dynamics etc. Aleatoric music or chance music comes from the Latin word alea meaning dice. This term was introduced to composers in the beginning of the 1950s by acoustian Werner Meyer- Eppler in a series of lectures. Mozart investigated composing through the use of statistics. He developed a method of composing by using certain musical measures and tossing of the dice to select a particular measure. These games consisted of a sequence of musical measures, for which each measure had several possible versions, and a procedure for selecting the precise sequence based on the throwing of a number of dice [6.]

With the advent of computers, developing mathematical music theory using statistics became popular. One of the earliest work in relating statistics to music is by Hiller in 1956. Hiller [7, 8] created the Illiac Suite later called the string quartet No. 4 which consisted of four movements created through four different experiments. In the fourth part of the composition, he used probability and stochastic processes to create musical piece. Hiller's work is generally regarded as the first ever score to be composed on an electronic computer.

According to Liu and Selfridge-Field [9], the process of musical composition can be modeled as a realization of an underlying random process, and the underlying random process is what we fuzzily call "musical styles". This was demonstrated by Fucks [10] who conducted experiments using statistics in music with empirical data. He identified certain musical parameters and used the mean and variance of these parameters for a sample of symphonies to analyze their musical styles. His study found significant differences in musical styles in terms of variance of the fourth order or kurtosis in the symphonies.

Around the 50s some path breaking work in this area was developed by Greek-French composer Xenakis. Infact Xenakis was a student of Oliver Messiaen, a great composer and organist who was known for using mathematics to create structural symmetries in his musical language [11]. Xenakis [12,13] pioneered the use of mathematical models like game theory and stochastic processes in Music which later paved the way for development of non-deterministic algorithmic music. His book 'Formalized Music: Thought and Mathematic in Composition', is regarded as an important contribution in the field of applications of stochastic processes, game theory and computer programming in music. He used these techniques in many of his compositions notable among them being *Metastaseis*an orchestra piece of 61 musicians. Xenakis was instrumental in establishing the school Equipe de Mathématiqueet Automatique Musicales (EMAMu) also known as Center for Mathematical and Automated Musicin 1966 for study of computer-assisted composition. In 1966 he won the Swedish Polar Music Prize in recognition of his efforts to popularize algorithmic music.

A statistical method that can be applied to music compositions is Stochastic processes. Stochastic processes include use of probability functions and Markov Chains to generate music. This paper discusses stochastic processes in relation to music composition. This section will present a preliminary discussion on generating stochastic patterns and the next section presents a review of Markov Chains and their applications in music composition.

3. STOCHASTIC PROCESSES AND STOCHASTIC MUSIC

Families of random variables which are functions of time are known as stochastic processes [14].The set of possible values or events of a single random variable taking random values of a stochastic process $\{ X_n, n\geq 1 \}$ is known as its state space. The state space is discrete if it contains a finite number of points, otherwise it is continuous[14]. A random variable in music composition could be defined as a musical parameter like pitch, dynamics or duration. The set of total events that each variable or parameter can assume form the state space E. Thus,

$$E = \{e_1, e_2, \ldots\ldots , e_i, \ldots., e_n\}$$

where, e_i represents the i^{th} value of the parameter and it can assume a total of n values. n represents the order of the state space.

For example if dynamics, pitch and duration are three parameters, their state spaces may be defined as:

E_{dyn} = {pp, p, mp, mf, f, ff}, E_{pitch} = {the major scale of C}, E_{dur} = {crochet, quaver, semiquaver}

The simplest case when all outcomes are equally likely results in an aleatoric process, a process in which each outcome is equally likely. A combination of events from the three spaces will give the parameter space. For each note a combination may be chosen at random using random number generation technique. With many parameters being controlled and with careful definition of each state space, the resulting sound over short periods will sound far from chaotic and may produce pleasing and original patterned sequences [15].

In practice, certain events may have a higher likelihood of occurrence than other events. In such cases, random distribution with weights assigned to the events in terms of probabilities can be used. The events that are more likely to occur are assigned a higher probability of occurrence, leading to what can be called stochastic music. Xenakis one of the pioneers who adopted a pure mathematical approach to music composition, is generally considered to have coined the term 'Stochastic Music'.

A probability assignment can be used in a computer program to generate a random sequence of notes forming a melody. This method can be modified by defining each event in the space stochastically making it more powerful. This will be discussed in the next section.

Conversely, it is also possible to analyze musical pieces by assigning probabilities for various events in a piece and counting the number of occurrences of each event. This opens up the possibility of analyzing, studying and comparing musical pieces. Jones [15] has used these techniques to analyze musical pieces by Handel and Bach. According to Jones, probability assignments can be useful in composition as long as the limitations are understood. A further improvement of this technique can be achieved by using probability distributions like Binomial, Poisson and Normal distributions. Jones [15] successfully used these techniques in a musical orchestra piece called 'Firelake'.

3.1 MARKOV CHAINS: A REVIEW OF APPLICATIONS IN MUSIC

Markov chains are named after the Russian Mathematician A. A. Markov. Markov developed this subject while analyzing the first 20,000 letters of Pushkin's 'Eugene Onegin' by studying the vowels and consonants. His findings and study led to the development of a new area in Probability theory called Markov Chains, now an integral part of the discipline of statistics.

A stochastic process is called a Markov Chain if the process undergoes transitions from one state to another on a state space, with the conditional probability distribution of the next state depending only on the current state and not on the sequence of events that preceded it. Mathematically, for any j,k, if p_{jk} is the conditional probability of transition from state j at the $(n-1)^{th}$ trial to the state k at the n^{th} trial then,

$$p_{jk} = P\ (\ X_n = k/\ X_{n-1} = j)$$

p_{jk}'s are non- negative and sum up to unity. They are called the transition probabilities and are central to the study of Markov Chains. These probabilities can be presented in the form of a matrix called transition probability matrix (TPM).

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \cdots\cdots \\ p_{21} & p_{22} & p_{23} \cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & & \end{pmatrix}$$

P is a stochastic matrix, i.e. a square matrix with non-negative elements and unit row suns. Markov chains are now extensively used in many diverse areas such as biology, physics, economics and engineering to name a few. Beran [16] explains the motivation of using Markov Chains in music in the following words, 'Musical events can often be classified into a finite or countable number of categories that occur in a temporal sequence. A natural question is then whether the transition between different categories can be characterized by probabilities. In particular, a successful model may be able to reproduce formally a listener's expectation of "what happens next", by giving appropriate conditional probabilities. Markov chains are simple models in discrete time that are defined by conditioning on the immediate past only'. Hiller made the use of Markov Chains in stochastic music compositions popular after he used them in the fourth part of his composition 'Illiac Suite" in 1956.Since then Markov Chains have been used by composers in music in various ways. Among its applications, it has been used for creating new music and composer classification or identification i.e. to classify or identify music compositions which are not known otherwise. In order to create new music, researchers first analyze sample sequences and then use the results to generalize and create new structures with similar characteristics as the sample structures. Brooks et. al. [17] were one of the first researchers from Harvard University to experiment with Markov Chains for creating new sequences of music. They took a sample of 37 melodies from various composers and conducted a detailed analysis using Markov Chains of up to the eight order for prediction. Liu and Selfridge-Field [9]gave a general framework to build up a Markov chain model to identify a music style in the following steps:

- Step 1: Define the repertoire of that style.
- Step 2: Encode all the works of that repertoire.
- Step 3: Define the state space S that consists of musical events.
- Step 4: From all the encoded works available in that repertoire, calculate the matrix P of conditional event-transition probabilities.

- Step 5: Let C be the Markov chain corresponding to {P, S}. Call C a Markov chain model for that particular style of music.

Based on this framework they developed a scheme for modeling music using Markov chains and applied it to two-way composer identification. Experiments were conducted by examining certain transitions of Haydn's and Mozart's string quartets, and the results showed that, as far as the recognition rate is concerned, the computer's performance was statistically significant. Wołkowicz, Kulka, and Kešelj [18] have conducted experiments for composer classification by using rhythmic and melodic features using Markov Models. Pollastri, E., &Simoncelli, G. [19], also used Markov models for composer identification. They studied a data set of 605 musical themes written by five well-known composers Mozart, Beethoven, Dvorak, Stravinsky and the Beatles. They then conducted experiments with Markov Chains for abstracting the style of a composer and for recognizing an unknown excerpt. Ames [20, 21] has conducted a detailed analysis of work related automated compositions from 1956 to 1986.Some more workon Markov Chains and their application in music are by McAlpine, K., et. al.. [22], Pearce, M. T., et. al., [23], Lejaren A. Hiller and Leonard M. Isaacson [24], Verbeurgt K., et. al. [25], Pearce and Wiggins [26].

An interesting application of Markov chains has been given by Wadi [27]. His paper presents a method based on Markov chains and linear algebra to measure the distance between two similar sounding musical pieces. They have applied the method to analyze similarities between the two songs 'ice ice baby' by Vanilla Ice and 'under pressure' by Queen and David Bowie which were the subject of a well known copyright case. Chai and Vercoe [28] have used Hidden Markov Models to classify folk music from three different countries. The results from their study of Irish, German and Austrian folk music showed thatshows that the melodies of folk music do carry some statistical features to distinguish them. Bell, C [29] has given a method for algorithmic music composition using Markov Chains and Genetic Algorithms.

Recent research in use of stochastic processes in music include works by Fox R., Crawford R. [30] and Goienetxea I, et. al. [31].

4. CONCLUSION

This study presents a review of stochastic processes as applied to music with an emphasis on Markov Chains. It briefly traces the history of algorithmic music and presents a discussion of some of the important contributions of greatcomposers who have experimented with stochastic models like Hiller, Xenakis and Fuchs. Markov Chains in particular have been used by many composers in stochastic music composition. The paper also highlights some important applications of Markov models with respect to music composition. The review demonstrates that stochastic processes have been successfully applied in Western Classical Music to compose musical pieces and for composer identification. These techniques have tremendous possibilities and in my opinion can be used to study Indian Classical Music as well. For example, these may be used to scientifically study the transition of different traditional musical genres like khayal, dhrupad, thumri and ghazal from ancient times to its contemporary forms, to identify characteristic styles of different gharanas and how these have evolved over a period of time and also to identify musical compositions of unknown origins as belonging to a certain gharana or a composer. Future research possibilities also include comparative study of differences in styles in Carnatic and Hindustani music using Markov models. The possibilities are definitely challenging but very promising and offer an exciting area in terms of interdisciplinary research for statisticians and musicologists. A major challenge of applying Markov Chains is that it is not always easy to define the state space and find the transition probabilities. For a monophonic piece of music defining the state space may be relatively simple. However, a polyphonic piece may consist of many instruments and voices and defining multiple state spaces may be challenging and require a more complex model. Also it is probably not correct to compare these methods with traditional methods of composition as algorithmic music can never replace traditional compositions. However these may be considered as an alternative method of composition and an artist may use it in a way that preserves the artistic sensibilities of the composer and the listener. Creating music using these methods may also lead to creation of new sounds. A potential area of future research in this direction will include an attempt to generate computer algorithms to create this music.

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