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Study Of Weather Analysis Via Geometric Modeling

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ABSTRACT

Weather phenomena such as sunbeams, clouds of all sorts, precipitation (ranging from light sprinkle to rain to hail and snow), fog, lightning, wind, humidity, and hot and cold conditions can all be part of much-larger-scale weather systems. The weather systems are cyclones (low pressure systems) and anticyclones (high-pressure systems) and the associated warm and cold fronts. The Weather condition of the atmosphere at a given time and place, usually expressed in terms of pressure, temperature, humidity, wind, etc. Also, the various phenomena in the atmosphere occurring from minutes to months. In this paper, we have defined and studied weather map analysis for numerical weather prediction via geometric modeling.

KEYWORDS: Geometric modeling, Weather phenomena, atmosphere, cyclones and anticyclones.

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1. INTRODUCTION

An important concept to hold is how weather patterns and the kinds of weatherthat occur relate to climate. We submit to this connectionas theweather machine because ofthe way the weather helps drives the climate system.It is the sum of many weather phenomenathat determines how the large-scale general circulationof the atmosphere. That is, the averagethree-dimensional structure (Geometric modeling) of atmosphericmotion.Actually works and it is the circulationthat essentially defines climate. This intimatelink between weather and climate provides abasis for understanding how weather eventsmay change as the climate changes.

We generally think of thefluctuations in the atmosphere from hour tohour or day to day as weather. Weather isdescribed by such elements as temperature, airpressure, humidity, cloudiness, precipitation ofvarious kinds, and winds. Weather occurs as awide variety of phenomena ranging from smallcumulus clouds to giant thunderstorms, fromclear skies to extensive cloud decks, from gentlebreezes to gales, from small wind gusts to tornadoes,from frost to heat waves, and from snowflurries to torrential rain. Many such phenomenaoccur as part of much larger-scale organizedweather systems which consist, in middle latitudes of cyclones (low pressure areas or systems) and anticyclones (high pressure systems) and their associated warm and cold fronts.Tropical storms are organized, large-scale systems of intense low pressure that occur in low latitudes. If sufficiently intense these become hurricanes, which are also known as typhoonsor tropical cyclones in other parts of the world.

2. WEATHER ANALYSIS

The glossary defines “analysis” as a comprehensive examination ofsomething. It is a process that breaks the object of theanalysis into parts so that each part can be examined in detail.This process determines its nature, function, and othercharacteristics. In operational weather analysis the object isthe atmosphere, particularly, the troposphere, where day-to-dayweather occurs. The parts that we need to examine are thevarious weather parameters that are naturally used to describethe atmosphere, temperature, humidity, wind speed and direction,atmospheric pressure, clouds, precipitation, etc.

We generally do our analysis at one specific time in order to get a glanceof what is happening in the atmosphere at that moment. However, analysis over time is often useful to provide a better pictureof how things are changing.In weather analysis and forecasting, I examined a wide varietyof weather data throughout the troposphere by integratingseveral observing systems during the weather analysis process.We have combine all this information into a three-dimensional image (geometric modeling) of what is producing the current weather by correlating what is observed with meteorological background.

Following tools are used in weather map analysis ***(Source: Surface weather analysis- Wikipedia.)***

Pressure Centers

Centers of surface (high-pressure and low-pressure) areas that are found within closed isobars ona surface weather analysis are the absolute maxima and minima in the pressure field,and can tell a user in a glance what the general weather is in their vicinity.

Low Pressure

Low-pressure systems, also known as cyclones, are located in minima in the pressurefield. Rotation is inmost at the surface and counterclockwise in the northernhemisphere as opposed to inmost and clockwise in the southern hemisphere due to the Coriolis force.

High Pressure

High-pressure systems, also known as anticyclones, rotate outward at the surface and clockwise in the northern hemisphere as opposed to outward and counter clockwise in the southern hemisphere.

Fronts

Fronts in meteorology are the leading edges of air masses that have a density, air temperature, and humidity different from the air mass it is invading. When an air mass passes over an area, it is marked by changes in temperature, moisture, wind speed and direction, atmospheric pressure, and often a changein the precipitation pattern. The change is called a front, whether it is warm or cold.

Cold Front

A cold front's location is at the leading edge of the temperature drop-off, which in an isotherm analysis shows up as theleading edge of the isotherm gradient, and it normally lies within a sharp surface trough. Cold fronts can move up to twiceas fast as warm fronts and produce sharper changes in weather, since cold air is denser than warm air and rapidly lifts thewarm air as the cold air moves in.

Warm Front

Warm fronts mark the position on the Earth's surface where a relatively warm body of air has displaced colder air. Thetemperature increase is located on the equatorward edge of the gradient in isotherms, and lies within broader low pressuretroughs than is the case with cold fronts. Warm fronts move more slowly than do the cold fronts because cold air is denser,and harder to displace from the Earth's surface.

Occluded Front

An occluded front is formed during the process of cyclogenesis when a cold front over takes a warm front. The cold and warm fronts curve naturally pole ward into the point of occlusion, which is also known as the triple point inmeteorology.

Stationary Fronts and Shear lines

A stationary front is a non-moving boundary between two different air masses, neither of which is strong enough to replace the other. They tend to remain inthe same area for long periods of time, usually moving in waves. There isnormally a broad temperature gradient behind the boundary with more widely spaced isotherms. A wide variety of weather can be found along a stationary front, but usually clouds and prolonged precipitation are found there.Stationary fronts will either dissipate after several days or devolve into shear lines, but can change into a cold or warmfront if conditions aloft change causing a driving of one air mass or the other.

Dry Line

The dry line is the boundary between dry and moist air masses east of mountain ranges with similar orientation to the Rockies, depicted at the leading edge of the dew point, or moisture, gradient. Near the surface, warm moist air that isdenser than warmer, dryer air wedges under the drier air in a manner similar to that of a cold front wedging under warmer air. When the warm moist air wedged under the drier mass heats up, it becomes less dense and rises and sometimes forms thunder storms.

3. NUMERICAL WEATHER PREDICTION MODEL

Since the early 20th century numerical weather prediction (NWP) has increasingly becomeone of the most important and complicated problems of modern science. With the advent ofcomputers, increased observations, and progress in theoretical understanding, numericalmodels were developed. Since then, such models are playing an increasing role in understandingand predicting weather and climate and have been a driving force in the advancementof the meteorological sciences. Numerical models are a mathematical representation ofthe earth's climate system including the atmosphere, ocean, cryosphere and land, amongothers. The models divide the area of interest into a set of grids and then make use of observationsof variables such as surface pressure, winds, temperature and humidity at numerouslocations throughout the globe. The observed values are then assimilated and used bythe model to predict future evolution of the earth's weather and climate. In the mid 20th century,models evolved from a simple model with a single atmospheric layer to a multi-layerprimitive equation model capable of predicting cyclone development (Stensrud,2007).Numerical Weather Prediction (NWP) is the science of predicting the weatherusing models of the atmosphere and computational techniques. Current weatherconditions are used at the input of the mathematical models of the atmosphere to predictthe weather.

Weather and climate models are based on the Navier-Stokes equations and moist thermodynamics,but substantial insights into the processes and mechanisms behind ubiquitous featuresof weather were obtained long before computing power reached the levels we take for grantedtoday. These insights were obtained by applying systematic approximations to the Navier-Stokes equations, while retaining important properties such as conservation laws for energy andpotential vorticity. Many decades of careful research have resulted in a deep understanding ofwhat lies behind the choreography of weather, and this understanding is utilized in the designof numerical models. By incorporating this knowledge via elegant numeric's, we focus on thefeatures that inuence the weather for the week ahead, rather than expending computer poweron all the eddying local gusts of wind that play little or no part in the evolution of the bigpicture.

We shall use the (Geometric Modeling) one-dimensional water equations as a vehicle to describe the salient mathematicalideas that help meteorologists to quantify some of the large-scale, often highly predictable,features of the atmospheric circulation. For brevity, we ignore thermodynamics in thedescription that follows, but the model is readily augmented to include the physics of heatand moisture (which are necessary if considering phenomena such as weather fronts), whileretaining the key mathematical structures.

The one-dimensional equations (Geometric Modeling) on a domain of R^2, with local Cartesian coordinates (x,y), and rotating with constant angular frequency $f/2$, are

$$\frac{Du}{Dt} - fv + g\frac{\partial h}{\partial x} = 0, \quad \frac{Dv}{Dt} + fu + g\frac{\partial h}{\partial y} = 0, (3.1)$$

$$\frac{Dh}{Dt} + h\nabla.u = 0,(3.2)$$

Where t denotes time, g is the acceleration due to gravity (a constant) and h(x, y, t) is the depth of the fluid. The horizontal velocity has two components u(x, y, t) = (u, v), and the Lagrangian, or material, derivative is $\frac{D}{Dt} = \frac{\partial}{\partial t} + u.\boldsymbol{\nabla}$.(3.3)

The Lagrangian derivative represents the change of a dependent variable as we "follow the flow"; i.e. it is a directional along the trajectory of a fluid particle. The derivative u.$\boldsymbol{\nabla}$ is often referred to as the advection, or transport, term. If any variable A(x, t) satisfies $DA/Dt = 0$, then we say that A(x,t) is conserved in the Lagrangian sense. Such conservation laws are of fundamental importance in metrology and oceanography. The so-called semi-geostrophic equations amount to following approximation to the shallow water equations

$$\frac{Du_g}{Dt} - fv + g\frac{\partial h}{\partial x} = 0, \quad \frac{Dv_g}{Dt} + fu = g\frac{\partial h}{\partial y} = 0,(3.4)$$

$\frac{Dh}{Dt} + h\nabla . u = 0$,(3.5)

Where u_g and v_g are two components of the geostrophic wind$g\nabla h = (fv_g, -fu_{g)}$.

The geostrophic flow is parallel to constant h (in the context of the Navier –Stokes equations on a rotating domain, the geostrophic flow is parallel to the isobars of constant pressure).The difference between the shallow water equations and the semi-geostrophic equations is the replacement of the fluid velocity (u, v) with the geo9strophic wind (u_g, v_g) in the Lagrangian derivatives of u and v, while leaving the derivatives operator (3.3) and the continuity equation (3.5) unchanged. This is known as the geostrophic momentum approximation (Hoskins 1975), and it applies to flows in which the rate of change of momentum is much smaller than the Coriolis force. The adverting velocity (that is, the velocity appearing in the Langrangian derivative u.$\boldsymbol{\nabla}$) is not approximated in semi-geostrophic flows, such as jet streams and fronts, have two distinct length scale (for example, a weather front is a relatively sharp discontinuity between air masses, but fronts extend to distances of the order of 1000km along the interface between air masses). In essence, the geostrophic momentum approximation tells us that it is important to represent the adverting velocity accurately; while the quantity is beingadverted (e.g. the geostrophic wind) can be approximated.

Equations (3.4) and (3.5) conserve energy and the following form of potential vorticity

$q = \frac{1}{h}(f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} + \frac{1}{f}\frac{\partial(u_g,v_g)}{\partial(x,y)})$. (3.6)

In atmospheric dynamics (with thermodynamics included), potential vorticity is proportional to the vector dot product of vorticity and stratification that, following the flow, can only be changed by diabatic or frictional processes. Potential vorticity is a fundamental concept for understanding the generation of vorticity in cyclogenesis (the birth and development of a cyclone), especially along the polar front, and in analyzing flow in the ocean. The use of potential vorticity played a key role as a diagnostic in understanding the evolution of Hurricane Sandy.

The semi-geostrophic equations have played a major role in understanding the formation of fronts(front genesis), and the properties of the equations that facilitate such ctudies are revealed through a transformation of coordinates. Defining new coordinates (sometimes called geostrophic momentum coordinates)

$\boldsymbol{X} \equiv (X, Y) \equiv (x + v_g, y - u_g)$(3.7)

We find that (4) may be replaced by

$\frac{DX}{Dt} = u_g \equiv (u_g, v_g)$.(3.8)

Hence the motion in these transformed coordinates is exactly geostrophic. The Jacobian is proportional to the potential vorticity (3.6)

$\frac{\partial(X,Y)}{\partial(x,y)} = \frac{hq}{f}$ (3.9)

The vector X may be expressed as the gradient of a scalar function P(x),

$X = \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial Y}\right)$, (3.10)

Which, to within an arbitrary additive constant, is uniquely defined by

$P(x,t) = \frac{1}{2}(x^2 + y^2) + \frac{gh(x,t)}{f^2}$. (3.11)

We note, using $g\nabla h = (fv_g, -fu_g)$ together with (3.7) and (3.11), that (3.9) has the form of a Monge-Ampere equation for P, given q(x, y, t) and suitable boundary conditions

$q = \frac{f}{h}\left(P_{xx}P_{yy} - P_{xy}^2\right)$. (3.12)

When the Jacobian (9) is non-singular, we express x(X) by introducing a scalar function R(X):$x = \left(\frac{\partial R}{\partial X}, \frac{\partial R}{\partial Y}\right)$, (3.13)

Where R is given to within an additive constant by

$R(X) = x.X - P(x)$ (3.14)

(Time is a parameter in this transformation). Equation (3.14) is the expression for the Lagendre transformation between R(X) and p(X). Local singularities of this map can be interpreted as atmospheric fronts (Chynoweth and Sewell 1989).

The semi-geostrophic equations can be integrated in time using the conservation of potential vorticity, expressed in geostrophic momentum coordinates. We write the reciprocal of the potential vorticity, q^{-1} as

$q(x,t)^{-1} \equiv \rho(X,t) = \frac{h(x,y)}{f}\left(R_{XX}R_{YY} - R_{XY}^2\right)$ (3.15)

This may be expressed solely in terms of the geostrophic momentum coordinates by defining $\text{ø}(x,t) = gh(x,t)/f^2$and $\Phi(X,t) = \frac{1}{2}(X^2 + Y^2) - R(X,t)$, then we note

$$\frac{\partial \Phi}{\partial X} = \frac{\partial \text{ø}}{\partial x} = X - x, \frac{\partial \Phi}{\partial Y} = \frac{\partial \text{ø}}{\partial y} = Y - y.$$

Hence (3.15) may be written

$$\rho(X,t) = \frac{f}{g}(\Phi - \frac{1}{2}(\Phi_X^2 + \Phi_Y^2)) \mid Hess(\frac{1}{2}(X^2 + Y^2) - \Phi \mid,$$

Where Hess (·) is the hessian matrix of the second derivatives of (·) with respect to X,Y.

We can show that $\boldsymbol{\rho}$ satisfies

$\frac{\partial \rho}{\partial t} + \dot{X}\frac{\partial \rho}{\partial X} + \frac{\partial \rho}{\partial Y} = 0$ (3.16)

Where $\dot{X} = f\left(\frac{\partial R}{\partial Y} - Y\right)$. $\dot{Y} = -f(\frac{\partial R}{\partial X}$ - X). (3.17)

The integration begins by solving the dual Monge-Ampere equation (3.15) for R, and then using R in (3.17) can be expressed in Hamiltonian form (McIntyre and Roulstone 2002):

$$\dot{X} = -f\frac{\partial \Phi}{\partial Y}, \dot{Y} = f\frac{\partial \Phi}{\partial X}.$$

The semi-geostrophic equations involve two important types of geometry, symplectic geometry associated with the Hamiltonian structures and contact geometry associated with the Legendre transformation. These two geometries also play a fundamental role in the theory of the Monge-Ampere equation (Kushner et al. 2007) and, in turn, relate to complex geometries, such as Kaehler geometry. The transformation properties of the semi-geostrophic equations can be understood within the context of hyper-Kahlar geometry, and this geometry has been exploited in the study of a much broader class of models of cyclones (McIntyre and Roulstone 2002, Delahaies and Roulstone2010).

The emergence of a complex structure, which at first sight is somewhat surprising given the classical nature of the fluid mechanics, occurs when the Monge-Amphee equation (3.12) is elliptic. This equation is elliptic when $q > 0$.the ellipticity is related to convexity condition on the energy and to the stability of the flow (Cullen 2006). The total energy, which is conserved by the shallow water equations(3.1), (3.2), is

$$E = \int (\tfrac{1}{2}(u^2 + v^2) + \tfrac{1}{2}gh)hdxdy. \quad (3.18)$$

This is a function of u, v and h, and the conditions for E to be minimized corresponds to the stability of a geostrophic flow, viewed as a solution of the unapproximated equations, to perturbations of the form

$$\delta u = f\delta y, \delta v = -f\delta x, \nabla.(\delta x, \delta y) = 0. \quad (3.19)$$

The second variation of E is greater than zeros when the Hessian matrix of the Legendre function P is positive definite. This corresponds to the ellipticity of the Monge-Ampere equation (3.2).Introducing momentum coordinates, (X', Y'), via $(u, v) \equiv f(y - Y', X' - x)$, we can show that the perturbations (3.19) imply $\delta\sigma = 0$, where

$$\sigma = h\frac{\partial(x,y)}{\partial(X',Y')}. \quad (3.20)$$

If we define s distance $d(x, X')$ between x and X' such that

$$d^2 = f^2((X' - x)^2 + (Y' - y)^2),$$

Then the energy functional can be rewritten

$$E = \int (\tfrac{1}{2}d^2(x, X') + \tfrac{1}{2}gh)hdxdy. \quad (3.21)$$

The proof of E can be uniquely minimized is established by showing that, given σ as a non-negative function of the momentum coordinates, there is a unique mapping from(X', Y') to (x,y) that minimizes E and satisfies (3.20). This is the starting point for expressing the energy minimization as a Monge optimal mass transport problem (Benamou and Brenier 1998; Culllen 2006; Villani 2008). Utilizing theorems on the regularity of solutions of the Monge-Ampere equation, together with optimal mass transport theory, it has been shown that the semi-geostrophic equations can be integrated for large times from suitable initial data. (Cullen and Roulstone, 1993) showed that a semi-geostrophic finite element numerical simulation of an Eady wave- the basic building block of cyclogenesis could be performed through multiple life-cycles, and the predictability of the key features of the system were remarkably robust to perturbations of the initial data.

4. CONCLUSION

Since the first forecast, the complexity and sophistication of numerical weather predictionModels have increased tremendously. The continued improvement in data assimilationand numerical models, and the continued availability of ever larger and faster computershave allowed numerical weather prediction to become more accurate. Most of the improvements in numerical models that occurred over the past 50 years can be categorized asimproved numerical techniques, improved model resolution, or improved modelphysics. Development of different types of grids, grid-staggering and grid-spacing are intimatelyrelated to the improvement in the weather and climate prediction.

The target of numerical weather simulations is to calculatethe state of atmosphere depending on time. That means thesimulation has the purpose to calculate the velocity, density,pressure, temperature and humidity of every single point inthe air. As it is not possible to regard every singlepoint because of observational and computational limitationsa two-dimensional or a three-dimensional manifolds is used forthe ability of the NWP model to accurately represent atmospheric phenomena is based onthree conditions; scientific knowledge, the availability of observational data, and computerprocessing abilities. If enough observational data is available and enough scientific knowledge is present then the limiting factor of an accurate forecast is the power of the processing computer.

REFERENCES

1. Bauer, P., Thorpe, A., Brunet, G., “The quiet revolution of numerical weather prediction. Nature”, 2015; 525: 47-55
2. Chynoweth, S., Sewell, M.J., “Dual variables in semi-geostrophic theory”. Proc. R. Soc. Lond., 1989; 424:155-86

3. Cullen, M.J.P., “A mathematical theory of large-scale atmosphere/ocean ow”.Imperial College Press, 2006.
4. Cullen,M.J.P., Norbury, J., Purser, R.J., Shutts, G.J. “Modelling the quasi-equilibrium dynamics of the atmosphere”. Q.J.R. Meteorol. Soc., 1987; 113: 735-57
5. Cullen, M.J.P., Roulstone, I., “A geometric model of the nonlinear equilibration of two dimensional Eady waves”. J. Atmos. Sci., 1993; 50: 328-32
6. Delahaies, S.B., Roulstone, I.,“Hyper-Kahler geometry and the semi-geostrophic Equations”. Proc. R. Soc. Lond., 2010; 466: 195-211
7. Hoskins, B.J. , “The geostrophic momentum approximation and the semi-geostrophic Equations”. J. Atmos. Sci., 1975; l32: 233-42
8. Kushner, A., Lychagin, V., Rubtsov, V.N., “Contact geometry and nonlinear deferential Equations”. Cambridge University Press, 2007.
9. McIntyre, M.E., Roulstone, I., “Are there higher-accuracy analogues of semi-geostrophic theory”? In Large-scale atmosphere {ocean dynamics, Volume 2, Norbury & Roulstone (eds.)}, 2002
10. Roulstone, I., Norbury, J., “Computing Superstorm Sandy: the mathematics of predictinga hurricane's path”. Scientic American, August, 2013; 309(2): 22
11. Villani, Cedric, “Optimal transport: old and new”. Springer, 2008.
12. Visram, A., Cotter, C.J., Cullen, M.J.P., “A framework for evaluating model error using Asymptotic convergence in the Eady model”. Q.J.R. Meteorol. Soc., 2014; 140: 1629-39
13. Human Intelligence. Francis Galton. (http://www.indiana.edu/~intell/galton.shtml) Retrieved on 2007-04-18.
14. Frank Rives Millikan. Smithsonian Institution. Joseph Henry: “Father of the Weather Service”. (http://www.si.edu/archives/ihd/jhp/joseph03.htm) Retrieved on 2006-10-22.
15. David M. Schultz. “Perspectives on Fred Sanders's Research on Cold Fronts“(http://www.cimms.ou.edu/~schultz/sanders/sanders.pdf), 2003, revised, 2004, 2006; 5- 8.
16. David Roth. Hydrometeorological Prediction Center. “Unified Surface Analysis Manual”. (http://www.wpc.ncep.noaa.gov/sfc/UASfcManualVersion1.pdf) Retrieved on 2006-10-22.
17. Saseendran S. A., Harenduprakash L., Rathore L. S. and Singh S. V. “A GIS application for weather analysis and forecasting”. (http://www.gisdevelopment.net/application /environment /conservation/envm0004.htm) Retrieved on 2007-05-05.
18. “National Weather Service”. Station Model Example.(http://www.srh.noaa.gov/ohx/educate /station_model.gif) Retrieved on 2007-04-29. Archived.

19. "Weather Doctor. Weather's Highs and Lows: Part 1 The High". (http://www.islandnet.com /~see/weather/elements/high.htm)
20. "BBC Weather. Weather Basics - Low Pressure". (http://www.bbc.co.uk/weather/features/ basics_lowpressure.shtml)Retrieved on 2007-05-05.
21. "BBC Weather. High Pressure". (http://www.bbc.co.uk/weather/features/understanding/ highpressure.shtml) Retrieved on 2007-05-05.
22. "United Kingdom School System. Pressure, Wind and Weather Systems". (http://atschool.eduweb.co.uk/radgeog/metlink/ppt/highs/Press-System.ppt) Archived (https://web.archive.org/web/20070927190000/http://atschool.eduweb.co.uk/radgeog/metlink /ppt/highs/Press-System.ppt) 2007-09-27 at the Wayback Machine.Retrieved on 2007-05-05.
23. "UniversityofIllinois.OccludedFront".(http://ww2010.atmos.uiuc.edu/(Gl)/guides/mtr/af/frnts /ofdef.rxml) Retrieved on2006-10-22.
24. "University of Illinois. Stationary Front". (http://ww2010.atmos.uiuc.edu/(Gl)/guides/mtr /af/frnts/sfdef.rxml) Retrieved on 2006-10-22.
25. "University of Illinois. Dry Line: A Moisture Boundary". (http://ww2010.atmos.uiuc.edu/(Gl)/guides/mtr/af/frnts/dfdef.rxml) Retrieved on 2006-10-22.
26. Stensrud, D. J. "Parameterization Schemes Keys to Understanding Numerical Weather Prediction Models". Cambridge University Press, (2007).
27. Lynch, P. "The Origins of Computer Weather Prediction and Climate Modeling". Journal of Computational Physics, 2008; 3431-3444.
28. J. Knievel, "Numerical Weather Prediction (NWP) and the WRFModel," 2006, Jason Knievel at the ATEC Forecasters Conference [https://www.ral.ucar.edu/projects/armyrange/ references/forecastconf 06/02 wrf. pdf] [last access: 21st June 2015].