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Solution of Integral Equation of Matrix Variable Involving Laguree Function

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ABSTRACT

The aim of present paper is to give an Inversion Formula for Fractional integral operator involving laguree function of Matrix Variable as

$$L_\nu^\gamma(T)=\frac{\Gamma_p(\gamma+\nu+\frac{p+1}{2})}{\Gamma_p(\gamma+\frac{p+1}{2})}\int_{0<U<T}\left|T-U\right|^{\gamma}{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}})f(U)dU$$

$$,\mathrm{Re}(\gamma,\nu)>(p-1)/2$$

by successive application of Laplace and inverse Laplace transform of Matrix Variable where $L_\nu^\gamma(T)$ is know, f(U) is to determined, T and U are real positive, definite and symmetry matrices of order p×p, and |U|=det U, is determinate of U.

All the matrices considered are real positive, definite and symmetric matrices of order p×p.at last we prepare table for different value of function $L_\nu^\gamma(T)$

KEY WORDS: Laplace transforms, integral equation, matrix variable

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INTRODUCTION

1. *Laplace and inverse Laplace transforms*

Laplace and inverse Laplace transforms of matrices variable are respectively given by equations

$$L_Z[f(Z)] = \int_{T>0} e^{tr(-TZ)} f(T)dT = \Phi(Z)$$

..........(1.1)

And

$$\frac{2^{p(p-1)/2}}{(2\pi i)^{p(p+1)/2}} \int_{\mathrm{Re}(Z)>0} e^{tr(TZ)}\Phi(Z)dZ = \begin{matrix} f(T), T>0 \\ 0, elsewhere \end{matrix}$$

...........(1.2)

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Where $\Phi(Z)$ is complex analytic function and integral is taken over Z=X+iY with X with fixed $X > X_0$ and Y over the space $S_p{}^*, S_p{}^*$ is the corresponding space of Z.

For the condition and detail the readers are referred to Mathai[1] and Mathai saxena[2]. Mathai.A.M and H.J. Haubold[3]

Mathai[1] generalized the Convolution theorem for Laplace transform for scalar varible to the Matrix Variables follows:
let $g_1(T)$ and $g_2(T)$ be Laplace transform of $f_1(X)$ and $f_2(X)$ respectively; then

$$L_T[\int_{0<U<X} f_1(X-U)f_2(U)dU] = g_1(T)g_2(T) \qquad . \qquad ....(1.3)$$

Where $\int_{0<U<X} = \int_0^X$ means that the integral is over U>0, X>0,X-U>0 and 0 denotes the null matrix of order p×p.

Sharma [4,5,6] obtain the solution of Integral Transform of Matrix Variable involving gauss hyper geometric Function , generalized stieltjes transform of Matrix variable.
The aim of present paper to give inversion formula for integral operator of Matrix variable using Laplace transform (1.1) and inverse Laplace transform (1.2).

2. *INTEGRAL EQUATION OF MATRIX VARIABLE*

Consider the convolution fractional integral operator involving Jacobi Polynomials

$$L_\nu^\gamma(T)=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}\int_{0<U<T}|T-U|^{\gamma}\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}})f(U)dU$$

$$,\mathrm{Re}(\gamma,\nu)>(p-1)/2 \quad \text{.....(2.1)}$$

Apply Laplace Tansform L_X (1.1) ; to both side of (2.1); we have

$$L_X[L_\nu^\gamma(T)]=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}L_X[\int_{0<U<T}|T-U|^{\gamma}\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}})f(U)dU\,]$$

$$,\mathrm{Re}(\gamma,\nu)>(p-1)/2$$

. ………………(2.2)

Using, Convolution theorem given by Mathai [1], page259(5.1.13)of Laplace transform

$$L_X[L_\nu^\gamma(T)]=\Gamma_P(\gamma+\nu+\frac{p+1}{2})\left|I+A^{\frac{1}{2}}X^{-1}A^{\frac{1}{2}}\right|^{\nu}|X|^{-\gamma-\frac{(p+1)}{2}}L_X[f(U)]$$

………(2.3)

$$\frac{L_X[g_\nu^\gamma(T)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}\quad|\mathrm{A}+\mathrm{X}|^{-\nu}|X|^{\nu+\gamma+\frac{(p+1)}{2}}\quad=\quad L_X[f(U)]$$

………..(2.4)

Taking Inverse Laplace Transform; we get

$$f(U)=L_U^{-1}[\frac{L_X[g_\nu^\gamma(T)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}\quad|\mathrm{A}+\mathrm{X}|^{-\nu}|X|^{\nu+\gamma+\frac{(p+1)}{2}}\;]$$

. ...(2.5)

Verification In order to verify (2.5) if we take

$$L_\nu^\gamma(T)=etr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})|T|^{-\nu-\frac{p+1}{2}}\quad,\mathrm{T}>0$$

……..(2.6)

Then first we take Laplace Transform of (2.6) is given as

$$L_X[etr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})|T|^{-\nu-\frac{p+1}{2}}]=\Gamma_P(-\nu)|A+X|^{\nu}$$

……(2.7)

Which is easily obtain by using Laplace transform and the inverse Laplace transform L_U^{-1} of (2.7) can be obtained by

$$L_U^{-1}\left[\frac{|X|^{\nu+r+\frac{p+1}{2}}\Gamma_P(-\nu)]L_X[L_\nu^\gamma(T)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}\right]$$

$$=\frac{|U|^{-\nu-r-(p+1)}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})}$$

… …..(2.8)

With $L_\nu^\gamma(T)$ as in (2.6), our solution of(2.1) is then

$$f(U)=\frac{|U|^{-\nu-r-(p+1)}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})} \quad ,\mathrm{U}=\mathrm{U}^1>0,\mathrm{Re}(\nu)>0$$

… ……(2.9)

Putting f(U)given by (2.9) in equation(2.1),we get

$$L_\nu^\gamma(T)=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}\int_{0<U<T}|T-U|^\gamma\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}})\frac{|U|^{-\nu-r-(p+1)}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})}dU$$

u sin *g lemma* 1*givenbyMathai*[1, *page*293(5.2.32)], *we* get

$$=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}\int_{0<U<T}|U|^\gamma\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}UA^{\frac{1}{2}})\frac{|T-U|^{-\nu-r-(p+1)}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})}$$

$,\mathrm{Re}(\gamma,\nu)>(p-1)/2$

. ...(2.10)

Using Mathai and Saxena [2], after the transformations

$U=T^{\frac{1}{2}}VT^{\frac{1}{2}}, dU=|T|^{\frac{1+p}{2}}dV$ *forFixed T* and the Respective Region is$0<\mathrm{U}<\mathrm{T}, 0<\mathrm{V}<\mathrm{I}$, we have

$$=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(\gamma+\frac{p+1}{2})\Gamma_P(-\nu)}{\Gamma_P(\gamma+\frac{p+1}{2})\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\nu)}etr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})|T|^{-\nu-\frac{p+1}{2}}$$

… ……..(2.11)

We have

$$L_\nu^\gamma(T)=etr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})|T|^{-\nu-\frac{p+1}{2}} \quad ,\mathrm{T}>0 \qquad \ldots(2.12)$$

Particular Case: if we take Matrix of order 1×1 i.e if we take p=1 we get,

$$f(u)=\frac{|U|^{-\nu-r-2}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+1)\Gamma_P(-\gamma-\nu-1)} \qquad ,U=U^1>0,\mathrm{Re}(\nu)>0$$

… …..(2.13)

Gives inverse of

$$L_\nu^\gamma(t)=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}\int_{0<U<T}|t-u|^\gamma\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-a^{\frac{1}{2}}(t-u)a^{\frac{1}{2}})f(U)dU$$

$,\mathrm{Re}(\gamma,\nu)>(p-1)/2...$

.

…….(2.14)

TABLE FOR DIFFERENT MATRIX VARIABLE

S. N		
	$L_\nu^\gamma(T)=\frac{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}{\Gamma_P(\gamma+\frac{p+1}{2})}\int_{0<U<T}\|T-U\|^\gamma\,{}_1F_1(-\nu,\gamma+\frac{p+1}{2}:-A^{\frac{1}{2}}(T-U)A^{\frac{1}{2}})f(U)dU$	$f(U)=L_U{}^{-1}[\frac{L_X[g_\nu^\gamma(T)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})}\quad\|\mathrm{A}+\mathrm{X}\|^{-\nu}\|X\|^{\nu+\gamma+\frac{(p+1)}{2}}]$
1	$\mathrm{e}\,tr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})\|T\|^{-\nu-\frac{p+1}{2}}$	$\frac{\|U\|^{-\nu-r-(p+1)}\Gamma_P(-\nu)]}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})}\quad ,\mathrm{U}=\mathrm{U}^1>0,\mathrm{Re}(\nu)$
2	$\mathrm{e}\,tr(-A^{\frac{1}{2}}TA^{\frac{1}{2}})\|T\|^{-\beta-\frac{p+1}{2}}$	$\frac{\|U\|^{-\beta-r-(p+1)}\Gamma_P(-\beta)]}{\Gamma_P(\gamma+\beta+\frac{p+1}{2})\Gamma_P(-\gamma-\beta-\frac{(p+1)}{2})}\quad ,\mathrm{U}=\mathrm{U}^1>0,\mathrm{Re}(\beta)$
3	$L_\nu^\gamma(T)$	f(U)
4	$\|T\|^{\nu-\frac{p+1}{2}}$	$\frac{\|U\|^{-\nu-r-(p+1)}\Gamma_P(\nu)]_1F_1(\nu;\nu+\gamma+\frac{p+1}{2};-AU)}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})},$ $R(\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-\nu-\frac{(p+1)}{2})>0$
5	$\|T\|^{\nu-\frac{p+1}{2}}\,{}_1F_1(\beta,\nu;-AU)$	$\frac{\|U\|^{-2\nu-r-(p+1)}\Gamma_P(\nu)_1F_1(\nu+\beta;2\nu+\gamma+\frac{p+1}{2};-AU)}{\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-2\nu-\frac{(p+1)}{2})}$ $\Gamma_P(\gamma+\nu+\frac{p+1}{2})\Gamma_P(-\gamma-2\nu-\frac{(p+1)}{2})$

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