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On the Mahgoub Transform and Ordinary Differential Equation with Variable Coefficient

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ABSTRACT:

The Mahgoub Transform, whose fundamental properties are presented in this paper .Here we apply new integral transform named as “Mahgoub Transform” to solve some ordinarydifferential equation with variable coefficient.

KEYWORDS : Mahgoub Transform – Differential Equation

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INTRODUCTION

Integral Transform play an important role in many fields of science .In literature, integral transforms is widely used in physics, astronomy, and optics and engineering mathematics.

The term "Differential Equation" was proposed in 1676 by Leibniz. The first studies of these equations were carried out in the late 17^{th}century. Differential equations are a powerful tool in the study of many problems in the science and in technology.

Integral Transform method is widely used to solve the several differential equations with the initial values or boundary conditions [1-5].

Recently Mohand.M.Mahgoub introduces a new integral transform named the "Mahgoub Transform"[6-8] and it has further applied to the solution of ordinary and partial differential equations. The purpose of this paper is to solve differential equations with variable coefficients using Mahgoub Transform.

Definition: Mahgoub Transform. A new transform called the Mahgoub transform defined for function of exponential order we consider function in the set a defined by

$$A = \left\{ f(t): \exists M, K_1, K_2 > 0, |f(t)| < M e^{\frac{|t|}{kj}} \right\}$$

For a given function in the set A, the constant M must be finite number, K_1, K_2 may be finite or infinite .Mahgoub transform which is defined by the integral equation.

$M[f(t)] = H(v) = v\int_0^\infty f(t)e^{-vt}dt, \quad t \geq 0, \ K_1 \leq v \leq K_2.$

Mahgoub Transform of some function:

$$M[1] = 1 M[t] = \frac{1}{v} M[t^n] = \frac{n!}{v^n}$$

$$M[e^{at}] = \frac{v}{v-a} M[\text{sinat}] = \frac{av}{v^2+a^2} M[\text{sinhat}] = \frac{av}{v^2-a^2}$$

$$M[\text{cosat}] = \frac{v^2}{v^2+a^2} M[\text{coshat}] = \frac{v^2}{v^2-a^2}$$

Theorem: If Mahgoub transform of the function f(t) given by $M[f(t)] = H(v)$ then,

(i) $M[tf(t)] = -\frac{d}{dv}H(v) + \frac{1}{v}H(v).$

(ii) $M[tf'(t)] = -\frac{d}{dv}[vH(v) - vf(0)] + \frac{1}{v}[vH(v) - vf(0)].$

(iii) $M[t^2 f'(t)] = v\frac{d^2H(v)}{dv^2} + 2\frac{dH(v)}{dv}.$

(iv) $M[tf''(t)] = -\frac{d}{dv}[v^2H(v) - v^2f(0) - vf'(0)] + \frac{1}{v}[v^2H(v) - v^2f(0) - vf'(0)].$

(v) $M[t^2 f''(t)] = v^2\frac{d^2H(v)}{dv^2} + 4v\frac{dH(v)}{dv} + 2\frac{dH(v)}{dv} - 2f(0).$

Proof: (i) $M[f(t)] = H(v) = v\int_0^\infty f(t)e^{-vt}dt$

$$\frac{d}{dv}H(v) = H'(v) = \frac{d}{dv}v\int_0^\infty f(t)e^{-vt}dt$$

$$= \frac{d}{dv}v\int_0^\infty f(t)e^{-vt}dt$$

$$= \int_0^\infty \frac{d}{dv}(ve^{-vt})f(t)dt$$

$$= \int_0^\infty (ve^{-vt})(-t)f(t)dt + \int_0^\infty f(t)e^{-vt}dt$$

$$= -v\int_0^\infty e^{-vt}[tf(t)]dt + \frac{1}{v}v\int_0^\infty f(t)e^{-vt}dt$$

$$\frac{d}{dv}H(v) = -M[tf(t)] + \frac{1}{v}H(v)$$

$$M[tf(t)] = -\frac{d}{dv}H(v) + \frac{1}{v}H(v)$$

To prove (ii) we use

$$M[tf(t)] = -\frac{d}{dv}H(v) + \frac{1}{v}H(v)$$

$$M[tf(t)] = -\frac{d}{dv}\big[M[f(t)]\big] + \frac{1}{v}\big[M[f(t)]\big]$$

Now we put $f(t) = f'(t)$ we have

$$M[tf'(t)] = -\frac{d}{dv}\big[M[f'(t)]\big] + \frac{1}{v}\big[M[f'(t)]\big]$$

$$M[tf'(t)] = -\frac{d}{dv}[vH(v) - vf(0)] + \frac{1}{v}[vH(v) - vf(0)].$$

To prove (iv) we use

$$M[tf(t)] = -\frac{d}{dv}\big[M[f(t)]\big] + \frac{1}{v}\big[M[f(t)]\big]$$

Now we put $f(t) = f''(t)$ we have

$$M[tf''(t)] = -\frac{d}{dv}\big[M[f''(t)]\big] + \frac{1}{v}\big[M[f''(t)]\big]$$

$$M[tf''(t)] = -\frac{d}{dv}[v^2H(v) - v^2f(0) - vf'(0)] + \frac{1}{v}[v^2H(v) - v^2f(0) - vf'(0)].$$

(iii) $M[t^2f'(t)] = -\frac{d}{dv}\left\{-v\frac{dH(v)}{dv} - H(v) + f(o)\right\}$

$$M[t^2f'(t)] = v\frac{d^2H(v)}{dv^2} + 2\frac{dH(v)}{dv}$$

(v) $M[t^2f''(t)] = -\frac{d}{dv}\left\{-v^2\frac{dH(v)}{dv} - 2vH(v) + 2vf(0) + f'(0)\right\}$

$$M[t^2f''(t)] = v^2\frac{d^2H(v)}{dv^2} + 4v\frac{dH(v)}{dv} + 2\frac{dH(v)}{dv} - 2f(0).$$

Now we apply the above theorem to find the Mahgoub Transform for some differential equations.

Example.1: Solve the differential equation with variable coefficient.

$$y'' + ty' - y = 0. \qquad y(0) = 0, \quad y'(0) = 1$$

Solution: Taking Mahgoub transform to given equation

$$[v^2H(v) - vf'(0) - v^2f(0)] - \frac{d}{dv}[vH(v) - vf(0)] + \frac{1}{v}[vH(v) - vf(0)] - H(v) = 0$$

$$[v^2H(v) - v] - [vH'(v) + H(v)] + H(v) - H(v) = 0.$$

$$H'(v) + H(v)\left(\frac{1 - v^2}{v}\right) = -1.$$

Which is linear differential equation .Its solution is

$$H(v) = \frac{1}{v} + Cv^{-1}e^{\frac{v^2}{2}}$$

We know $y(0) = 0$,then C=0

$$H(v) = \frac{1}{v}$$

By using inverse Mahgoub transform

$$y = t$$

Example: 2. Consider the ordinary differential equation with variable coefficient s

$$ty'' - ty' + y = 2. \qquad y(0) = 2, \quad y'(0) = -1$$

Solution: Taking Mahgoub transform of given equation

$$-\frac{d}{dv}[v^2H(v) - vf'(0) - v^2f(0)] + \frac{1}{v}[v^2H(v) - vf'(0) - v^2f(0)]$$

$$+\frac{d}{dv}[vH(v) - vf(0)] - \frac{1}{v}[vH(v) - vf(0)] + H(v) = 2$$

This gives

$$H'(v) + H(v)\left(\frac{1 - v}{v - v^2}\right) = 2\left(\frac{1 - v}{v - v^2}\right)$$

$$H'(v) + H(v)\frac{1}{v} = \frac{2}{v}$$

This is linear differential equation. Its solution is

$$H(v) = 2 + \frac{C}{v}$$

By using inverse Mahgoub transform.

$$y = 2 + Ct$$

$y' = C$and$y'(0) = -1$ therefor $\mathrm{C} = -1$

$$y = 2 - \mathrm{t}$$

CONCLUSION:

In this paper we apply new integral transform "Mahgoub Transform" which is little know and not widely used to solve some ordinary differential equation with variable coefficient s, the result reveals that the proposed method is very efficient and simple and can be applied to linear and nonlinear differential equations.

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