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13 Optimizing Inventory and Marketing Policy under Two Ware-House Storage System for Deteriorating Itemswith Generalized Type Holding Cost Rates

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ABSTRACT

This paper considers an inventory system with deteriorating item under the two warehouse storage concept. In this paper the demand is a function of advertisement of an item and selling price. This paper assist the retailer in maximizing the total profit by determining the optimal inventory policy and marketing parameters. In contrast to previous inventory models, an arbitrary holding cost rate has been incorporated to provide general framework of the model. First a mathematical model is developed and the numerical examples are given to illustrate and validate the model applicability. Result analysis has been performed based on the decision parameters.

KEYWORDS: Two-warehouse, inventory, two components dependent demand rate, generalised type holding cost rates.

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INTRODUCTION

In recent years inventory problems for deteriorating items have been widely studied considering different type of deterioration rates under single ware house system and some of the researchers are using this phenomenon to develop an inventory system considering two-warehouse facilities. Most of the classical inventory models consider the demand of inventories is either constant or time dependent; many researchers have developed the model considering this type of demand pattern with various combination of business environment. Demands of certain type of products depend on the price fixed for selling the products. In a competitive market it is general trend that whenever selling price of an item reduces the demand increases and hence sales revenue increases. In essence, the lower selling price raises the selling rates whereas the higher selling price has reverse effect. Abad[1]has developed an inventory model considering price sensitive demand rates under single warehouse system. Several scholars have developed inventory models under consideration with price sensitive demand rates and single warehouse system such as Mukopadhyay[2]et.al.,Begam[3] et. al. and Sana[4].

Deterioration in the product is a common phenomenon in an inventory system and products are deteriorated during its normal storage period. The rate of deterioration depends on the nature of the product and the facilities provided in the warehouse. Some author considered that the utility of goods remains constant during its normal storage periods. But in general, certain product, during the storage period such as food, vegetables, fruits, medicine, blood, fish, alcohol, gasoline and radioactive chemicals deteriorates. The problem of deteriorating inventory has received considerable attention in recent years and to control and maintain the inventory of the deteriorating items to satisfy the demand of customers or retailers, it becomes an integral part of inventory system to study about deterioration. In the past few decades, many researches have drawn their attention towards the study of deteriorating items. Ghare and Shrader[5] were first to propose the inventory model where the factor of deterioration was considered on a two-warehouse inventory model and they observed the significant effect of deterioration on the inventory system. Later, there are many authors who use the phenomenon of deterioration with different type of deterioration rates to develop the inventory model considering single warehouse system such as Roy[6], and Covert and Philip[7]and so forth. Some authors studied inventory system under two storage facilities for deteriorating items such as Wee[8]et. al.,Yang[9],Sarma[10],Banarjee and Agrawal[11] and Yang[12] and so forth.

In the present scenario of business, advertisement of product is playing a crucial role in raising sales of product and is a major parameter affecting the marketing policy and it hasbe come the common trend to use the advertising policy to promote a product through print media, electronic media or other means to attract the customers. Considering the effect of price and advertising on demand rate various inventory models under single ware house inventory system have been developed by the researchers like Subramanyam and Kumaraswamy[13]and so forth.

In the single warehouse inventory system the limited storage is a major practical problem for real situation due to the lack of large storage space at an important market places, forcing retailers to own a small warehouse. However, to reduce the problem of storage retailers prefer to rent a house for a limited period. In case deteriorating items, specially equipped storage facility is required to reduce the amount of deterioration. To handle this situation the requirement of another storage space, providing the required facilities become necessity and therefore it become essential to use an additional warehouse on rental basis for a sort time period according to necessity of business. This rented warehouse is abbreviated as RW and the owned warehouse as OW while studying the two-warehouse inventory system. Generally, it is assumed that the rented ware house provides better storage facilities as compared to own ware-house and due to this the rate of deterioration in RW is smaller than OW which results more holding cost at RW there foreit is necessary to vacant RW first and thereafter the demand of customers is fulfilled from OW.

In, traditional models, holding cost is considered to be constant but in the present market scenario due to increasing demand of storage space the owner of rented warehouses may raise the holding cost after some time. Another factor of raising holding cost is due to providing better facilities in the warehouse to reduce the deterioration rate of the product and to maintain the constant deterioration rate throughout the time being utilised for storing products. Dye[14] has proposed a joint pricing and ordering policy for deteriorating items and N.H. Shah[15]et. al. as proposed an inventory model for non-instantaneous deteriorating items with generalised type deterioration and holding cost rates to optimize the inventory and marketing policy.

Based on above discussion and motivated by above papers, the proposed inventory model for deteriorating items under two-warehouse inventory system considers the demand rate depending upon the selling price and frequency of advertisement and the general type of holding cost rate with constant deterioration rate. The general type of holding rates assumes that up to a fix point of cycle length the constant holding cost and thereafter charge in the holding cost occurs. The model is developed with the objective of maximizing the profit of the retailers under above consideration and different cases depending on the time up to which holding cost is constant have been discussed separately. Numerical examples are provided to demonstrate the developed model and results are analysed.

The rest of the paper is organized as follows: In section 2, assumptions and notations are given which are used throughout the article. In section 3, mathematical model is developed to maximize the profit. In section 4 solution procedure is provided. Numerical examples are presented in section 5 and results are analysed in section 6. Finally, a conclusion is drawn in section-7.

ASUMPTIONS AND NOTATIONS

The mathematical model of the two-storage inventory problem is based on the following assumptions and notations:

2.1 *Assumptions*

- Replenishment rate is infinite and instantaneous.
- Storage capacity of RW is considered to be unlimited.
- The lead time is negligible and initial inventory level is zero.
- Shortages are not permitted.
- Demand rate $f(s,a)$ is a function of marketing parameters with the frequency of advertisement (a) and the selling price (s).In this paper, power form of selling price and the frequency of advertisement for demand function is considered; i.e. $f(s,a) = a^{\sigma}K\, s^{-b}$ where K (>0) is scaling factor, b (>1) is index of price elasticity, and σ is the shape parameter, where $0 \le \sigma < 1$.
- In both ware houses, deterioration is considered as constant parameter.
- Holding cost rates has taken generalised type in both warehouses.
- The deteriorated units cannot be repaired or replaced during the storage period.
- Deterioration occurs as soon as items are entered into inventory system.
- Inventory system considers a single item.

2.2 *Notations*

The following notations are used throughout the paper:

A	Ordering Cost
Q	Inventory level in RW at$t = 0$;
W	Capacity of OW.
α	Deterioration rate in RW; $\alpha > 0$;
β	Deterioration rate in OW; $\beta > 0$;
A_c	Cost for each advertisement.
p	Purchase cost per unit of item.

s	Selling price per unit.
$h_1(t)$	Holding cost per unit per unit time in RW at time t.
$h_2(t)$	Holding cost per unit per unit time in OWat time t.
Q_{max}	The maximum ordered quantity for a cycle length.
$I_r(t)$	Inventory level at any time t for r =1, 2, 3 etc. in RW and OW.
μ	Time at which inventory level in RW vanish.
T	Length of replenishment cycle.
γ	Time up to which holding cost is constant.
δ	Holding cost parameter
Π (s, a,μ,T)	Total profit per unit time of inventory system.

MATHEMATICAL MODEL

In the beginning of each cycle a Q_{max} units of items arrive in the inventory system. W units of items are stored in OW and rest items (Q) are stocked into RW. Depending on the values of γthree cases viz. $0 < \gamma \leq \mu$,$\mu < \gamma \leq T$ and $T < \gamma$ arise. Each case is discussed in details as follows:

Case-1: $\boldsymbol{0 < \gamma \leq \mu}$

The graphical representation for this case is shown in figure 1.In this case, the inventory level during the time interval $[0 \ \ \mu]$in RW decreasing to zero due to combined effect of demand and deterioration and the level of inventory in OW remain constant. Based on above information the status of inventory level at any time t is represented by differential equations

$$\frac{\mathrm{d}\, I_1(t)}{\mathrm{dt}} = -f(s,a) - \alpha\, I_1(t); \qquad 0 \leq t \leq \mu \qquad (1)$$

$$\frac{\mathrm{d}\, I_2(t)}{\mathrm{dt}} = -\beta I_1(t); \qquad 0 \leq t \leq \mu \qquad (2)$$

With boundary conditions$I_1(t) = 0$ at $t = \mu$ and $I_2(t) = W$.The solution of eqs. (1)and (2) is, respectively

$$I_1(t) = \frac{f(s,a)}{\alpha}\left(e^{\alpha(\mu-t)} - 1\right); \qquad 0 \leq t \leq \mu \qquad (3)$$

$$I_2(t) = We^{-\beta\, t} \qquad 0 \leq t \leq \mu \qquad (4)$$

In the beginning, inventory level in RW at $t = 0$ is Q i.e. $I_1(0) = Q$,using this condition in equation (3) the initial amount of inventory (Q) in RW is given by

$$Q = \frac{f(s,a)}{\alpha}(e^{\alpha\mu} - 1); \quad (5)$$

The total order quantity in each cycle is given by

$$Q_{max} = W + \frac{f(s,a)}{\alpha}(e^{\alpha\mu} - 1); \quad (6)$$

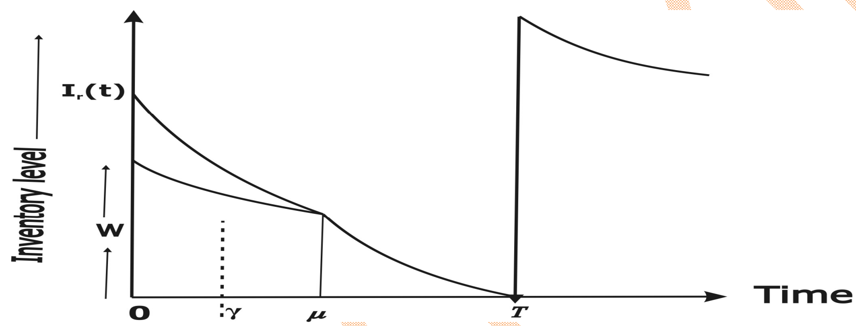

Figure-1: Graphical representation of inventory system for case-1

When level of inventory reaches to zero at $t = \mu$ in RW then, during time interval $[\mu \quad T]$the demand is fulfilled by supplying inventory from OW and therefore inventory decrease to zero due to combined effect of deterioration rate and demand. Based on above information the status of inventory level in OW at any time t is represented by differential equation

$$\frac{dI_3(t)}{dt} = -f(s,a) - \beta I_1(t); \qquad \mu \le t \le T \quad (7)$$

with boundary condition $I_3(t) = 0 \; at \; T = 0$, the solution of eq. (7) is

$$I_3(t) = \frac{f(s,a)}{\beta}\left(e^{\beta(T-t)} - 1\right); \qquad \mu \le t \le T \quad (8)$$

Thus the total present worth inventory cost during the cycle length, consist of the following costs elements

Ordering cost is A

The advertisement cost is $A_c * a$

Inventory holding cost in RW is

$$\int_0^{\gamma} h_1(t) I_1(t) dt + \int_{\gamma}^{\mu} h_1(t) I_1(t) dt$$

$$= \int_0^{\gamma} h_1(t) \frac{f(s,a)}{\alpha} \left(e^{\alpha(\mu-t)} - 1\right) dt + \int_{\gamma}^{\mu} h_1(t) \frac{f(s,a)}{\alpha} \left(e^{\alpha(\mu-t)} - 1\right) dt$$

Inventory holding cost in OW is

$$\int_0^{\gamma} h_1(t) I_2(t) dt + \int_{\gamma}^{\mu} h_2(t) I_1(t) dt + \int_{\mu}^{T} h_1(t) I_3(t) dt$$

$$= \int_0^{\gamma} h_2(t) W e^{-\beta t}\, dt + \int_{\gamma}^{\mu} h_2(t) W e^{-\beta t}\, dt + \int_{\mu}^{T} h_1(t) \frac{f(s,a)}{\beta} \left(e^{\beta(T-t)} - 1\right) dt$$

The purchase cost is $p * Q_{max}$

The sale revenue for a cycle is $s * f(s,a) * T$

Hence the average profitin the time interval $[0\ T]$ andfor this case, denoted by Π_1 (s, a, μ, T), isgiven by

$$\Pi_1(s, a, \mu, T) = \frac{1}{T}\begin{bmatrix} \text{Sales revenue} - \text{ordering cost} - \text{advertising cost} \\ -\text{holding cost} - \text{purchase cost} \end{bmatrix} \quad (9)$$

$$\Pi_1(s, a, \mu, T) = \frac{1}{T}\Big[sf(s,a)T - A - A_c\, a - \int_0^{\gamma} h_1(t) \frac{f(s,a)}{\alpha}\left(e^{\alpha(\mu-t)} - 1\right)dt - \int_{\gamma}^{\mu} h_1(t) \frac{f(s,a)}{\alpha}\left(e^{\alpha(\mu-t)} - 1\right)dt - \int_0^{\gamma} h_2(t) W e^{-\beta t}\, dt - \int_{\gamma}^{\mu} h_2(t) W e^{-\beta t}\, dt - \int_{\mu}^{T} h_1(t) \frac{f(s,a)}{\beta}\left(e^{\beta(T-t)} - 1\right)dt - p\, Q_{max}\Big] \quad (10)$$

Case-2: $\boldsymbol{\mu < \gamma \le T}$

The graphical representation for this case is shown in figure 2.In this case, holding cost rates remain constant till the inventory vanishes in RW and is increasesup to the end of cycle length and the average profit in the time interval $[0\ T]$, denoted by $\Pi_2(s, a, \mu, T)$ is given by

$$\Pi_2(s, a, \mu, T) = \frac{1}{T}\Big[sf(s,a)T - A - A_c\, a - \int_0^{\mu} h_1(t) \frac{f(s,a)}{\alpha}\left(e^{\alpha(\mu-t)} - 1\right)dt - \int_0^{\mu} h_2(t) W e^{-\beta t}\, dt - \int_{\mu}^{T} h_1(t) \frac{f(s,a)}{\beta}\left(e^{\beta(T-t)} - 1\right)dt - p\, Q_{max}\Big]; \quad (11)$$

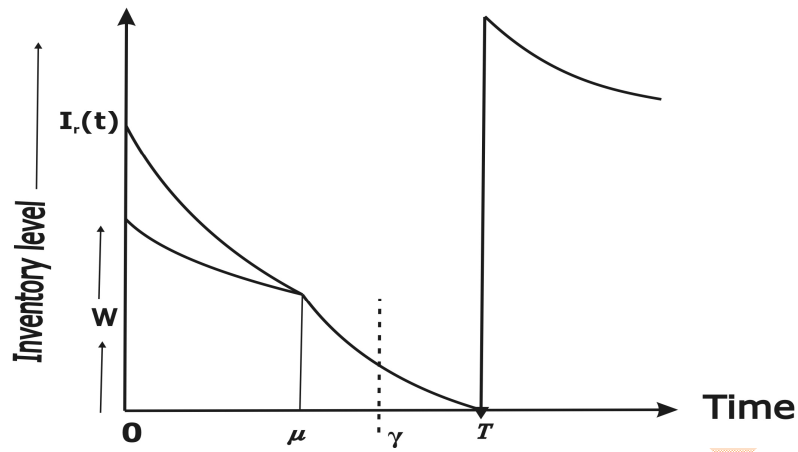

Figure-2: Graphical representation of inventory system for case-2

Case-3: $T < \gamma$

The graphical representation of this case is shown in figure 3. In this case, the time γ up to which holding cost rates remain constant for each cycle is more than the cycle length. Thus the model become traditional inventory model and the average profit in the time interval $[0\ T]$, denoted by $\Pi_3(s, a, \mu, T)$ is given by

$$\Pi_3(s, a, \mu, T) = \frac{1}{T}\Big[sf(s,a)T - A - A_c\ a - \int_\gamma^\mu h_1(t)\frac{f(s,a)}{\alpha}\left(e^{\alpha(\mu-t)} - 1\right)dt - \int_0^\mu h_2(t)We^{-\beta t}\ dt - \int_\mu^T h_1(t)\frac{f(s,a)}{\beta}\left(e^{\beta(T-t)} - 1\right)dt - p\ Q_{max}\Big]; \quad (12)$$

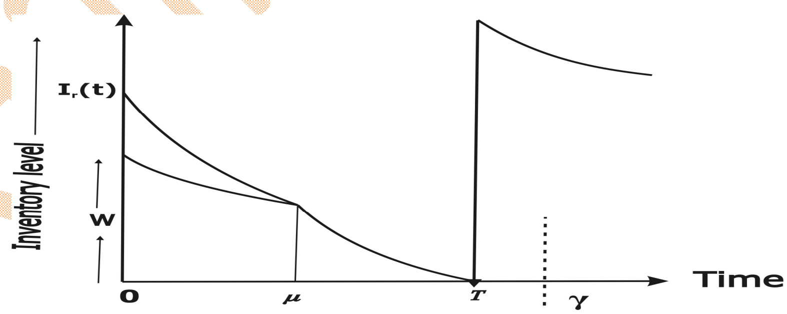

Figure-3: Graphical representation of inventory system for case-3

Thus the total profit of the inventory system is given by

$$\Pi(\mathrm{s,a},\mu,\mathrm{T}) = \begin{cases} \Pi_1(\mathrm{s,a},\mu,\mathrm{T}) & if\ 0 < \gamma \leq \mu \\ \Pi_2(\mathrm{s,a},\mu,\mathrm{T}) & if\ \mu < \gamma \leq T \\ \Pi_3(\mathrm{s,a},\mu,\mathrm{T}) & if\ T < \gamma \end{cases} \quad (13)$$

SOLUTION PROCEDURE

The necessary and sufficient condition to maximize the total average profit is

Maximize $\Pi(\mathrm{s,a},\mu,\mathrm{T})$

Subject to: $(s > 0, \mu > 0, T > 0)$

For known values of frequency of advertisement and selling price, the following condition must be satisfied to find the optimal solution

$$\frac{\partial \Pi(\mathrm{s,a},\mu,\mathrm{T})}{\partial \mu}=0; \frac{\partial \Pi(\mathrm{s,a},\mu,\mathrm{T})}{\partial \mathrm{T}} = 0; \quad (14)$$

and

$$\left(\frac{\partial^2 \Pi(\mathrm{s,a},\mu,\mathrm{T})}{\partial \mu^2}\right)\left(\frac{\partial^2 \Pi(\mathrm{s,a},\mu,\mathrm{T})}{\partial T^2}\right) - \left(\frac{\partial \Pi(\mathrm{s,a},\mu,\mathrm{T})}{\partial \mu\, \partial \mathrm{T}}\right)^2 < 0$$

Solving equation (14) the optimal value of decision variables can be obtained as μ^* and T^* and with these values the total average profit can becalculated using eq.(13)depending on the value of parameter γ.

NUMERICAL EXAMPLES

To illustrate the developed model, the holding cost function in RW and OW are considered respectively as

$$h_1(t) = \begin{cases} c_h^1 & if \quad t \leq \gamma \\ c_h^1 + \delta(t-\gamma) & if \quad t \geq \gamma \end{cases}$$

and

$$h_2(t) = \begin{cases} c_h^2 & if \quad t \leq \gamma \\ c_h^2 + \delta(t-\gamma) & if \quad t \geq \gamma \end{cases}$$

where $c_h^1 > 0$, $c_h^2 > 0$, $\delta > 0$

Example-1: In this example the following set of values of parameters are considered as $\mathrm{A} = \$250$ per order, $\mathrm{b} = 2$, $\mathrm{K} = 1000$, $\mathrm{a} = 1$, $\mathrm{s} = \$5$ per unit,

$\mathrm{p} = \$1$ per unit, $c_h^1 = \$0.6$ per unit per year, $c_h^2 = \$0.4$ per unit per year, $\mathrm{A_c} =$

\$80 per advertiesment, $\beta = 0.05$, $\sigma = 0.02$, $W = 100$, $\delta = 0.2$ .The results obtained from using the model for two cases 1 and 2aregiven in Table-1.

Table-1: "Values of Average profit"

Case	S	A	μ^*	T*	Q_{max}	Average profit ($)
1 ($\gamma = 3$)	5	1	7.7290	16.7461	90993.10	6894.56
		2	7.7337	16.7548	92696.30	6986.38
		3	7.7385	16.7629	93899.60	7038.74
	6	1	7.7412	16.7677	65414.80	4807.82
2 ($\gamma = 15$)	5	1	1.0655	20.7711	176.09	1035.60
		2	1.0655	20.7758	177.15	1046.49
		3	1.0655	20.7807	177.78	1051.36
	6	1	1.0655	20.7834	152.84	740.619

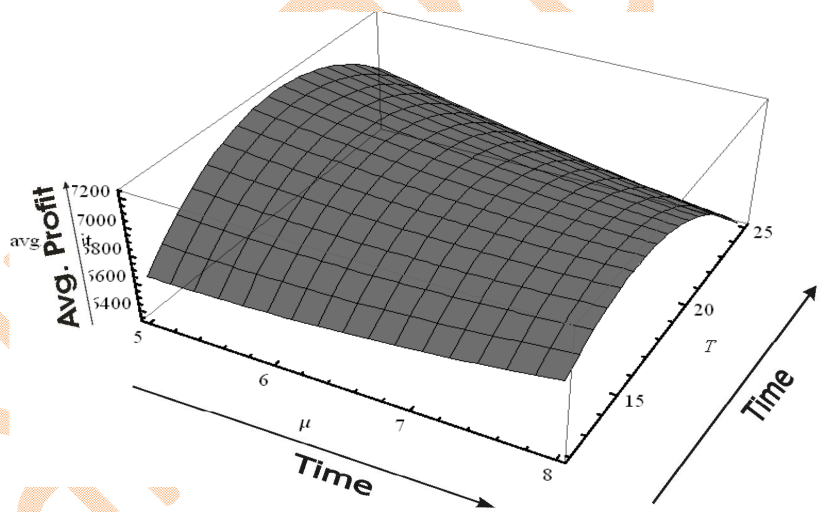

Figure-4: Graphical representation of inventory system for case-1

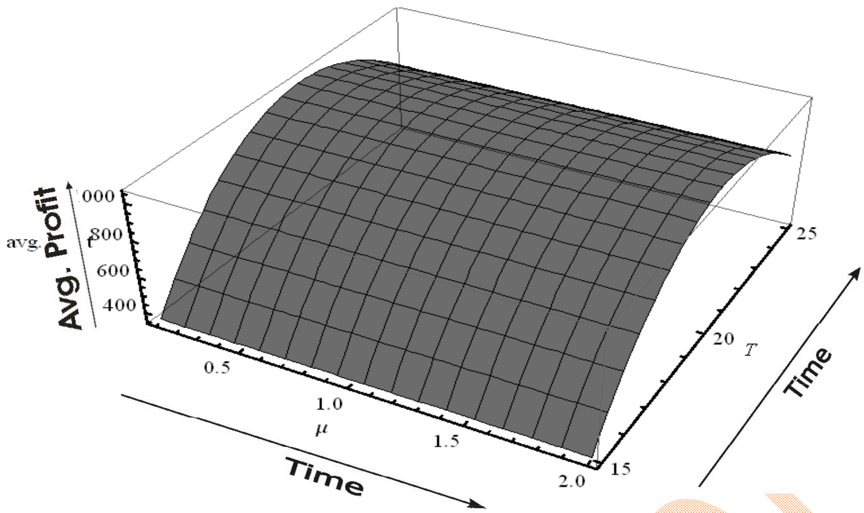

Figure-5: Graphical representation of inventory system for case-2

Example-2: In this example, same set of values of parameters are considered as given in example 1.Since in this case the model becomes traditional model as the holding cost rate is constant throughout the cycle length therefore $\delta = 0$.The result obtained using the model inthis case is given in Table-2.

Table-2: "Values of Average profit"

Case	s	A	μ^*	T*	Q_{max}	Average profit ($)
3	5	1	0.4766	6.4776	124.42	30.84
		2	0.4903	6.4856	125.66	19.87
		3	0.4982	6.4903	126.40	08.35
	6	1	0.0462	6.2439	101.31	28.42

RESULT ANALYSIS

From Table-1 and Table-2, the following observations are made:-

1) In case-1, for fixed value of s =5 and increasing number of advertisement, the length of cycle increases and profit is maximum at a=3.The profit decreases with increase in the value of selling price.
2) In case-2, for fixed value of s =5 and increasing number of advertisement length of cycle increases and profit goes to maximum level for a=3.The profit decreases with increase in the value of selling price.

3) In case-3, for fixed value of s =5 and increasing number of advertisement, the length of cycle increases and profit is maximum for a=1 and cycle length is lowest. The profit decreases with increase in the value of selling price.
4) Concavities of Profit function for case-1, depicted in Figure-4(μ^*, T^* verses average profit) for fixed values of $a = 3$ and $p = 5$.
5) Concavities of Profit function for case-2,depicted in Figure-5 (μ^*, T^* verses average profit) for fixed values of $a = 3$ and $p = 5$.
6) Concavities of Profit function for case-3depicted in Figure-6 (μ^*, T^* verses average profit)for fixed value of$a = 1$ and $p = 5$.
7) From Table-1 and Table-2, it is observed that the model is more profitable in case-2, as compared to case-1 and case-3.
8) It is also observed that increase in the frequency of advertisement increases the profit while increase in selling price lowers the profitin case-1 and case-2.

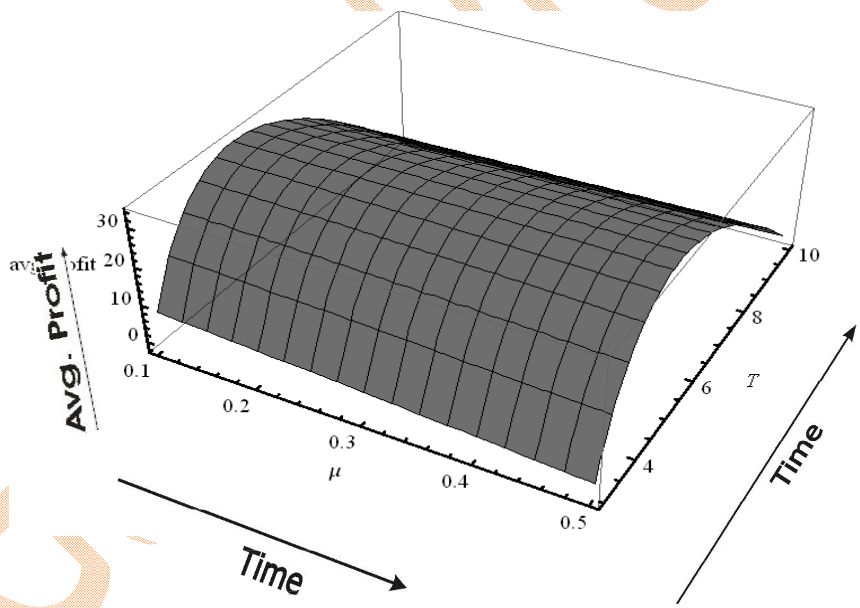

Figure-6: Graphical representation of inventory system for case-3

CONCLUSION

In this paper,a two-warehouse deterministic inventory model with constant deterioration rate and generalised type holding cost rateis developed to optimize the total profit of retailer. Price and advertisement dependent demand rateis evolved to decide the marketing policy for optimal solution. The optimization technique is used to derive the optimum replenishment policy and results are obtained using software Mathematical 9.0. Numerical examples are presented to illustrate the model efficiency. Higher profit is observed for hiring a warehouse on rental basis. Profit is found to be maximum, in case varying holding cost as compared to traditional model described in case-3. Thus the model is most useful when selling price is lowered, increases number of advertisement and inventory is purchased in bulk quantity and a warehouse is rented for storage. Further this paper can be enriched by incorporating other types of demand and generalized type deterioration rate in combination with generalized type holding cost and inflation.

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