Research Article **Available online** www.ijsrr.org **ISSN: 2279–0543**

International Journal of Scientific Research and Reviews

On the homogeneous equation of eighth degree with five unknowns $(x+y+z)^8 = (x+y)^4(w^2 - wt + t^2)^2$

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ABSTRACT

This paper focuses on finding different sets of non-zero distinct integer solutions to the homogeneous eighth degree equation with five unknowns given by $(x+y+z)^8 = (x+y)^4(w^2 - wt + t^2)^2$

KEYWORDS: Homogeneous eighth degree, Eighth degree with five unknowns, Integer solutions

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INTRODUCTION:

The theory of diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non - homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity[1,2,3]. Particularly in [4,5] special equations of sixth degree with four and five unknowns are studied. In[6,7,8,9] heptic equations with three, five and six unknowns are analysed. In[10,11,12,13,14,15] equations of eighth degree with four,five and six unknowns are analysed. This paper concerns with the problem of determining non-trivial integral solutions of the homogeneous equation of eighth degree with five unknowns given by $(x+y+z)^8=(x+y)^4(w^2-wt+t^2)^2$

METHOD OF ANALYSIS

The homogeneous eighth degree equation with five unknowns to be solved is

$$(x+y+z)^8=(x+y)^4(w^2-wt+t^2)^2 \quad (1)$$

Introducing the linear transformations

$$x=u+v, y=u-v, z=(2n-2)u, w=r+s, t=r-s, u\neq v, r\neq s \quad (2)$$

in (1), it leads to

$$r^2+3s^2=4n^4u^2 \quad (3)$$

Solving (3) through various ways and using (2), the corresponding integer solutions to (1) are obtained.

Way 1:

It is noted that (3) is satisfied by

$$r=2n^2(3A^2-B^2), s=4n^2AB, u=3A^2+B^2 \quad (4)$$

In view of (2),the corresponding integer solutions to (1) are given by

$$x=3A^2+B^2+v$$
$$y=3A^2+B^2-v$$
$$z=(2n-2)(3A^2+B^2)$$
$$w=2n^2(3A^2-B^2+2AB)$$
$$t=2n^2(3A^2-B^2-2AB)$$

Way 2:

Assume

$$u=a^2+3b^2, a, b\neq 0 \quad (5)$$

and write $4n^4$ as

$$4n^4 = (n^2 + i\sqrt{3}n^2)(n^2 - i\sqrt{3}n^2) \quad (6)$$

Substituting (5) and (6) in (3) and employing the method of factorization, define

$$r + i\sqrt{3}s = (n^2 + i\sqrt{3}n^2)(a + i\sqrt{3}b)^2 \quad (7)$$

Equating real and imaginary parts in (7), we get

$$\left.\begin{array}{l} r = n^2(a^2 - 3b^2 - 6ab) \\ s = n^2(a^2 - 3b^2 + 2ab) \end{array}\right\} \quad (8)$$

Substituting (5) &(8)in (2), the corresponding integral solutions of (1) are represented by

$$x = a^2 + 3b^2 + v$$
$$y = a^2 + 3b^2 - v$$
$$z = (2n - 2)(a^2 + 3b^2)$$
$$w = 2n^2(a^2 - 3b^2 - 2ab)$$
$$t = -8n^2 a\, b$$

Note 1:

In addition to (6), one may write $4n^4$ as exhibited below:

$$4n^4 = \frac{[(2 + i8\sqrt{3})n^2][(2 - i8\sqrt{3})n^2]}{49},$$

$$4n^4 = \frac{[(22 + i10\sqrt{3})n^2][(22 - i10\sqrt{3})n^2]}{196}$$

Following the above procedure, two more sets of integer solutions to (1) are obtained.

Way 3:

(3) is written as

$$r^2 + 3s^2 = 4n^4u^2 * 1 \quad (9)$$

Assume 1 on the R.H.S. of (9) as

$$1 = \frac{(1 + i\sqrt{3})(1 - i\sqrt{3})}{4} \quad (10)$$

Following the procedure as in Way 2 , the corresponding integer solutions to (1) are given by

$$x = a^2 + 3b^2 + v$$
$$y = a^2 + 3b^2 - v$$
$$z = (2n-2)(a^2 + 3b^2)$$
$$t = n^2(6b^2 - 2a^2 - 4ab)$$
$$w = -8n^2 ab$$

Note 2:

In addition to (10), one may write 1 as exhibited below:

$$1 = \frac{[(1+i4\sqrt{3})][(1-i4\sqrt{3})]}{49},$$
$$1 = \frac{[(11+i5\sqrt{3})][(11-i5\sqrt{3})]}{196}$$

Following the above procedure, two more sets of integer solutions to (1) are obtained.

Way 4:

(3) can be written in the form of ratio as

$$\frac{r+s}{2n^2u+2s} = \frac{2n^2u-2s}{r-s} = \frac{A}{B}, \text{where } B \neq 0 \qquad (11)$$

Solving (11) by applying the method of cross multiplication and using (2), the integral solutions of (1) are given by

$$x = 2(A^2 + B^2 - AB) + v$$
$$y = 2(A^2 + B^2 - AB) - v$$
$$z = (2n-2)(2A^2 + 2B^2 - 2AB)$$
$$w = 2n^2(-2A^2 + 4AB)$$
$$T = 2n^2(-2B^2 + 4AB)$$

Note 3:

(3) can also be written in the form of ratio as

$$\frac{2n^2u+r}{s} = \frac{3s}{2n^2u-r} = \frac{A}{B}, B \neq 0$$

For this choice, the corresponding integer solutions to (1) are given by

$$x = A^2 + 3B^2 + v$$
$$y = A^2 + 3B^2 - v$$
$$z = (2n-2)(A^2 + 3B^2)$$
$$w = 2n^2(A^2 - 3B^2 + 2AB)$$
$$T = 2n^2(A^2 - 3B^2 - 2AB)$$

CONCLUSION:

In this paper, we have obtained different patterns of solutions to Octic equation with five unknowns. As Diophantine equations are rich in variety, one may search for other choices of octic equations with multi variables for obtaining their integer solutions with suitable properties.

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