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**On Ingrained Construction of a Newfangled almost GP - Spaces in Simple Extended Topological Spaces**

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**ABSTRACT**

This paper elucidates about a new fangled form of the almost GP spaces under the ceiling of simple extension topological spaces. In addition it conveys the characteristics of almost P and almost GID spaces in simple extended topological spaces.

**KEYWORDS:** Meager<sup>+</sup> set, Residual<sup>+</sup> set, Almost P<sup>+</sup> spaces, Almost GP<sup>+</sup> spaces, Almost GID<sup>+</sup> - spaces.

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## 1. INTRODUCTION

In 1963, Levine<sup>3</sup> introduced the concept of simple extension of a topology  $\tau$  by a non open set  $B$  as  $\tau^+(B) = \{O \cup (O' \cap B) / O, O' \in \tau, B \notin \tau\}$  in simple extension as well. Let  $A$  be a subset of a space  $X$ . Then the closure of  $A$  is assumed in the extended topology and the interior of  $A$  is taken in general topology are denoted by  $cl^+(A)$  and  $int(A)$  respectively.

A subset of a topological space is a  $G_\delta$ -set if it is the intersection of countably many open sets; it is an  $F_\sigma$ -set if it is the complement of a  $G_\delta$ -set.

A completely regular space  $X$  in which every nonempty  $G_\delta$ -set has nonempty interior is called an almost P-space. Almost P-spaces was first introduced by A. I. Veksler<sup>4</sup> and it was also studied further by R. Levy in <sup>11</sup>. A  $T_1$  topological space  $X$  is called an almost GP-space (respectively a GID-space) if every dense  $G_\delta$ -set of  $X$  has nonempty interior (respectively dense interior).

Throughout this paper,  $(X, \tau^+)$  (or simply  $X$ ) represent a non-empty simple extended topological space (or simply extended topological space) on which no separation axioms are assumed, unless and otherwise mentioned.

## 2. SOME DISPOSITIONS OF DENSE, NOWHEREDENSE, MEAGER AND RESIDUAL SETS IN EXTENDED TOPOLOGICAL SPACES

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau^+)$  is said to be nowhere dense+ in  $X$  if  $int(cl^+(A)) = \emptyset$ , i.e the interior of the closure of  $A$  is empty. Otherwise put,  $A$  is nowhere dense+ iff it is contained in a closed set  $(\tau^+cl)$  with empty interior.

**Proposition 2.2:** Let  $X$  be an extended topological space. Then:

- (a) Any subset of a nowhere dense+ set is nowhere dense+.
- (b) The union of finitely many nowhere dense+ sets is nowhere dense+.
- (c) The closure of a nowhere dense+ set is nowhere dense+.

**Proof:** Obvious from the definition and the elementary properties of closure and interior.

**Definition 2.3:** A subset  $A \subseteq X$  is called meager+ (or of first category) in  $X$  if it can be written as a countable union of nowhere dense+ sets.

**Definition 2.4:** Any set that is not meager+ is said to be nonmeager+ (or of second category). The complement of a meager+ set is called residual+ set

**Definition 2.5:** Let  $I(X)$  be the set of all isolated points of  $X$ . If  $I(X) = \emptyset$ , then the space  $X$  is said to be crowded and a spaces is said to be separable if it contains a countable dense+ subset.

**Proposition 2.6:** Let  $X$  be a extended topological space. Then:

- (a) Any subset of a meager+ set is meager+.
- (b) The union of countably many meager sets is meager+.
- (c) If  $X$  has no isolated points, then every countable set is meager+.

The following lemma is useful in the sequel.

**Lemma 2.7:** Let  $A \subseteq X$  be a nowhere dense+ set. Then the closure of  $A$  is a nowhere dense+ set .

**Definition 2.8:** A subset  $A$  of a topological space  $(X, \tau^+)$  is said to be dense+ in  $X$  if  $\text{int}(\text{cl}^+(A)) = X$ .

### 3. ALMOST $P^+$ -SPACES, ALMOST $GP^+$ -SPACES AND ALMOST $GID^+$ -SPACES

**Definition 3.1:** The *almost- $P^+$ -spaces*, consists of those spaces in which  $G_\delta$ sets have dense+ interiors.

**Example 3.2:** Let  $X = \{a, b, c\}$  with the topology  $\tau = \{X, \emptyset, \{c\}\}$  and  $\tau^+ = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$ . here  $G_\delta$ sets =  $\{c\}$  and also  $\text{int}(\text{cl}^+(\{c\})) = \{c\}$ . And hence  $(X, \tau^+)$  is almost  $P^+$ - space.

Recall that completely regular spaces  $X$  in which every nonempty  $G_\delta$ -set of  $X$  has nonempty interior is called an almost  $P$ -space.

And hence a completely regular+ space  $X$  in which every nonempty  $G_\delta$ -set of  $X$  has nonempty interior is called an almost  $P^+$ -space.

**Definition 3.3:** A subset of a topological space is a  $G_{\delta^+}$ -set if it is the intersection of countably many  $\tau^+$ -open sets; it is an  $F_{\sigma^+}$ -set if it is the complement of a  $G_{\delta^+}$ -set.

In the following definition we give a generalization of almost  $P^+$ -spaces as follows:

**Definition 3.4:** A topological space  $X$  is called an almost  $GP^+$ -space if every dense+  $G_\delta$ -set of  $X$  has nonempty interior in  $X$ .

**Example 3.5:** Let  $X = \{a, b, c\}$  with the topology  $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$  and  $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ . Here  $\text{dense}^+ G_\delta \text{ sets} = \{a, b\}$  and also  $\text{int}(\{a, b\}) = \{a, b\}$ . And hence  $(X, \tau^+)$  is almost  $GP^+$ -space.

The following theorem gives the characterization of Baire space in SETS.

**Definition 3.6:** A topological space  $X$  is called an almost  $GP^{++}$ -space if  $\text{int}(V) \neq \emptyset$  for every dense $^+$  sets  $V$  and  $V = \bigcap_{n \in \mathbb{N}} V_n$ ,  $V_n$ 's are non-empty  $\tau^+$ -open sets in  $(X, \tau^+)$ .

In other words, A topological space  $X$  is called an almost  $GP^{++}$ -space if every dense $^+$   $G_\delta$ -set of  $X$  has nonempty interior in  $X$ .

**Theorem 3.7:** Suppose  $(X, \tau^+)$  be SETS. Then  $X$  is a Baire space iff every non-empty open set is of II-Category.

**Theorem 3.8:** If  $(X, \tau^+)$  be a SETS and  $T_1$  – crowded separable Baire space, then  $(X, \tau^+)$  is not a almost  $GP^+$ -space.

**Proof:** Let  $D$  be a countable dense $^+$  subset of  $X$ . Then  $X - D = \bigcup_{d \in D} \{X - \{d\}\}$ . Since  $X$  is Baire,  $X - D$  is a dense $^+$  set. Thus,  $X - D$  is dense $^+$  and  $X - D = \bigcup_{d \in D} \{X - \{d\}\}$  where  $\{X - \{d\}\}$ 's are open sets in  $X$ . Since  $D$  is dense $^+$  in  $X$ ,  $\text{int}(X - D) = \emptyset$ . Thus,  $(X, \tau^+)$  is not an almost  $GP^+$ -space.

**Theorem 3.9:** Let  $Y$  be an open dense $^+$  subset of  $(X, \tau^+)$ . Then  $Y$  is a almost  $GP^+$  Space iff  $X$  is almost  $GP^+$  Space.

**Theorem 3.10:** Let  $(X, \tau^+)$  and  $(Y, \sigma^+)$  be two SETS. If  $f : X \rightarrow Y$  is a continuous, feebly open, surjective mapping and  $X$  is a almost  $GP^+$  space, then  $Y$  is a almost  $GP^+$  space.

**Theorem 3.11:** Let  $(X, \tau^+)$  and  $(Y, \sigma^+)$  be two SETS. If  $f : X \rightarrow Y$  is a homeomorphism and  $Y$  is a almost  $GP^+$  space, then  $X$  is a almost  $GP^+$  space.

**Theorem 3.12:**  $\prod_{i \in I} X_i$  is a almost  $GP^+$  space iff each  $X_i$  is a almost  $GP^+$  space.

**Theorem 3.13:** Let  $\mathcal{U}$  be a collection of open subsets of a SETS  $X$  whose union is dense in  $X$ . If there is some  $U \in \mathcal{U}$  such that  $U$  is a almost  $GP^+$  space then  $X$  is a almost  $GP^+$  space.

**Theorem 3.14:** For a SETS  $X$ , the following conditions are equivalent.

- (a) For every  $V$ , if  $V$  is a dense set and  $V = \bigcap_{n \in \mathbb{N}} V_n$  where  $V_n$ 's are non-empty open sets in  $X$ , the  $\text{cl}^+(\text{int}(V)) = X$ .
- (b) Every nonempty open subspace of  $X$  is a almost  $GP^+$ -space.

**Definition 3.15:** A topological space  $X$  is called an almost  $GID^+$ -space if every dense<sup>+</sup>  $G_\delta$ -set of  $X$  has dense<sup>+</sup> interiors.

**Theorem 3.16:** If  $X$  is a (Baire) almost  $GID^+$ -space and  $D$  is dense<sup>+</sup> in  $X$ , then  $D$  is a (Baire) almost  $GID^+$ -space.

**Theorem 3.17:** For a SETS  $X$  the following conditions are equivalent:

- (1)  $X$  is a almost  $GID^+$ -space.
- (2) Every nonempty open subspace of  $X$  is a almost  $GID^+$ -space.
- (3) Every nonempty open subspace of  $X$  is an almost  $GP^+$ -space.

**Theorem 3.18:** Let  $X$  be an almost  $GID^+$ -space. If  $f: X \rightarrow Y$  is continuous, onto and open, then  $Y$  is a almost  $GID^+$ -space.

**Proposition 3.19:** Let  $U$  be a collection of open subsets of the space  $X$  whose union is dense in  $X$ . Then, every member of  $U$  is a  $GID$ -space, and then  $X$  is a  $GID$ -space.

**Corollary 3.20:** The topological sum of a family of almost  $GID^+$ -spaces (almost  $GP^+$ -spaces) is a almost  $GID^+$ -space (an almost  $GP^+$ -space).

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