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Fibonacci Prime Labeling of Udukkai and Octopus Graphs

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ABSTRACT:

A Fibonacci prime labeling of a graph $G = (V(G), E(G))$ with $|V(G)| = n$ is an injective function $g: V(G) \rightarrow \{f_2, f_3, \ldots . f_{n+1}\}$, where f_n is the n^{th} Fibonacci number that induces a function $g^*: E(G) \rightarrow N$ defined by $g^*(uv) = \gcd\{g(u), g(v)\} = 1 \ \forall\ uv \in E(G)$. The graph G admits a Fibonacci prime labeling is called a Fibonacci prime graph. In this paper we prove that udukkai graph and octopus graph are Fibonacci prime graphs.

KEYWORDS: Fibonacci prime graph, Udukkai graph, Octopus graph, duplication, fusing, switching.

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I.INTRODUCTION

In this paper, only finite simple undirected connected graphs are considered. The graph G has vertex set V =V (G) and edge set E = E (G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy[1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout[5].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[2] has proved that the path P_n on n vertices is a prime graph. Around 1980 Roger Entringer conjectured that all trees have prime labeling which has not been settled so far.

II.BASIC DEFINITIONS

Definition 2.1

The Fibonacci number f_n is defined recursively by the equations

$f_1 = 1$; $f_2 = 1$; $f_{n+1} = f_n + f_{n-1}$ $(n \geq 2)$.

Note 2.2

It is observe that, $g.c.d(f_n, f_{n+1}) = 1 \;\; \forall\, n \geq 1$,

$$g.c.d(f_n, f_{n+2}) = 1 \;\forall\, n \geq 1.$$

Definition 2.3

A prime labeling of a graph G is an injective function $f: V(G) \to \{1,2, \ldots .. |V(G)|\}$ such that for every pair of adjacent vertices $u \; and \; v$, $\gcd\{f(u), f(v)\} = 1$.A graph which admits a prime labeling is called a prime graph.

Definition 2.4

A Fibonacci prime labeling of a graph $G = (V, E)$ with $|V(G)| = n$ is an injective function $g: V(G) \to \{f_2, f_3, \ldots \ldots .. f_{n+1}\}$,where f_n is the n^{th} Fibonacci number that induces a function $g^* : E(G) \to N$ defined by $g^*\,(uv) = g.c.d\{g(u), g(v)\} = 1 \;\forall\, uv \in E(G)$.

The graph which admits a Fibonacci prime labeling is called Fibonacci prime graph.

Definition 2.5

An udukkai graph $U_{n,} n \geq 2$ is a graph constructed by joining two fan graphs $F_n, n \geq 2$ with two paths $P_n, n \geq 2$ by sharing a common vertex at the centre.

Definition 2.6

An octopus graph O_n $(n \geq 2)$ can be constructed by joining a fan graph F_n, $(n \geq 2)$ with a star graph $K_{1,n}$ by sharing a common vertex which is the centre of the star.

Definition 2.7

Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

Definition 2.8

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 2.9

Let u and v be two distinct vertices of a graph G.A new graph G_1 is constructed by identifying (fusing) two vertices u and v by a single vertex x in G such that every edge which was incident with either u or v in G now incident with x in G.

III. MAIN RESULTS

Theorem 3.1

An udukkai graph $U_n, n \geq 2$ is a Fibonacci prime graph where n is any positive integer.

Proof:

Let U_n be the udukkai graph with vertices $\{u_1, u_2, \ldots\ldots\ldots u_{4n-1}\}$, where u_1 is the common vertex .Here $|V(U_n)|=4n-1$.

Define $g: V(U_n) \to \{f_2, f_3 \ldots \ldots \ldots \ldots . f_{4n}\}$ as follows

$g(u_1) = f_2$

$g(u_i) = f_{i+1}, 2 \leq i \leq 4n-1$

Define the induced function $g^*: E(U_n) \rightarrow N$ by

$$g^*(uv) = g.c.d\{g(u), g(v)\} \; \forall \, uv \in E(U_n).$$

Clearly the vertex labels are distinct.

Now, $g.c.d\{g(u_1), g(u_i)\}$ = gcd $\{f_2, f_{i+1}\}$ =gcd $\{1, f_{i+1}\}$ = 1 for all i.

$$\text{g.c.d}\{g(u_i), g(u_{i+1})\} = \text{gcd}\{f_{i+1}, f_{i+2}\} = 1 \; for \; 2 \leq i \leq 4n-1.$$

Thus U_n admits Fibonacci prime labeling .Hence U_n is a Fibonacci prime graph.

Example 3.2

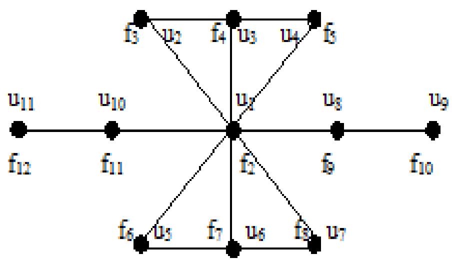

Figure 1: Fibonacci prime labeling for U_3

Theorem 3.3

The switching of an apex vertex u_1 in an udukkai graph U_n, $n \geq 2$ produces a Fibonacci prime graph, where n is any positive integer.

Proof:

Let $G = U_n$ and $V(U_n) = \{u_1, u_2 \ldots \ldots \ldots u_{4n-1}\}$ be the vertex set of an udukkai graph U_n ,$n \geq 2$.Let G_u denotes the graph obtained by an apex vertex switching of G with respect to the vertex u_1 .It is obvious that $|V(G_u)|$=4n-1.

Define $g: V(G_u) \rightarrow \{f_2, f_3 \ldots \ldots \ldots \ldots .. f_{4n}\}$

by $g(u_1) = f_2$

$$g(u_i) = f_{i+1}, 2 \leq i \leq 4n-1$$

Then the induced function $g^*: E(G_u) \rightarrow N$ is defined by

$g^*(uv)$ = gcd$\{g(u), g(v)\}$ $\forall$ $uv \in E(G_u)$

Now,gcd $\{g(u_1), g(u_i)\}$ = gcd $\{f_2, f_{i+1}\}$ = gcd $\{1, f_{i+1}\}$ = 1 for all i

gcd$\{g(u_i)$, $g(u_{i+1}\}$= gcd $\{f_{i+1}, f_{i+2}\}$ =1 for $2 \le i \le 4n-1$

Thus G_u admits a Fibonacci prime labeling .Hence G_u is a Fibonacci prime graph.

Illustration 3.4

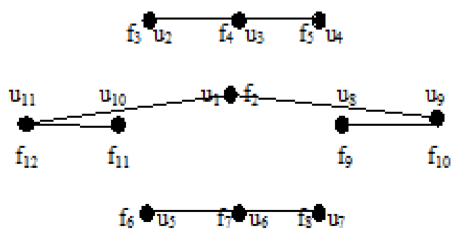

Figure 2: Switching an apex vertex u_1 in U_3 .

Theorem 3.5

An octopus graph O_n is a Fibonacci prime graph where n is any positive integer.

Proof:

Let G be an octopus graph O_n .Let $u_1, u_2 \ldots \ldots \ldots \ldots . u_{2n+1}$ be the vertices of G.Let E(G) be the edge set of an octopus graph where

$E(G) = \{u_1 u_i \ \ / \ \ 1 \le i \le 2n+1\} \cup \{u_i u_{i+1} \ \ / \ \ 2 \le i \le n\}$.

Here |V (G)|=2n+1, where n is any positive integer.

Define $g: V(G) \rightarrow \{f_2, f_3, \ldots \ldots \ldots \ldots . f_{2n+2}\}$

by $g(u_1) = f_2$

$g(u_i) = f_{i+1}, \ \ 2 \le i \le 2n+1.$

Then the induced function $g^*: E(G) \rightarrow N$ *is* defined by $g^*(uv)$ = gcd$\{g(u), g(v)\}$ $\forall uv \in E(G)$.

Now,gcd $\{g(u_1), g(u_i)\}$ =gcd $\{f_2, f_{i+1}\}$

=gcd $\{1, f_{i+1}\}$ =1 for $1 \leq i \leq 2n + 1$

gcd$\{g(u_i), g(u_{i+1})\}$ = gcd $\{f_{i+1}, f_{i+2}\}$ =1, $2 \leq i \leq n$

Clearly the vertex labels are distinct and

$g^*(uv)$ = gcd $\{f(u), f(v)\}$ =1$\forall uv \in E(G)$.

$\therefore G$ admits a Fibonacci prime Labeling .Hence G is a Fibonacci prime graph.

Example 3.6

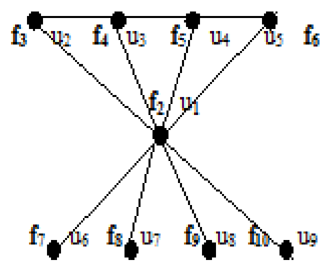

Figure 3: Fibonacci prime labeling for O_4

Theorem 3.7

The graph obtained by duplication of a pendant vertex u_k of an octopus graph O_n is a Fibonacci prime graph, where n is any positive integer.

Proof:

Let G be an octopus graph O_n. Let $u_k$be the pendant vertex of an octopus graph $O_n and\ u_k{}'$ be its duplicated pendant vertex. Let G_k be the graph obtained by duplication of the pendant vertex u_k in O_n ,where n is any positive integer.

Then |V (G_k)|=2n+2.

Define $g: V(G_k) \to \{f_2, f_3 \dots \dots \dots \dots \dots f_{2n+3}\}$ by

$g(u_1) = f_2$

$g(u_i) = f_{i+1}$ for $2 \leq i \leq 2n + 2$

Then the induced function $g^*: E(G_k) \rightarrow N$ is defined by

$g^*(uv) = \gcd\{g(u), g(v)\} \forall uv \in E(G_k)$

Now,gcd $\{g(u_1), g(u_i)\} = \gcd(f_2, f_{i+1})$ =1 for all i.

$\gcd\{g(u_i), g(u_{i+1})\} = \gcd(f_{i+1}, f_{i+2})$ =1$\forall u_i u_{i+1} \in E(G_k)$.

Clearly vertex labels are distinct and $g^*(uv) = \gcd\{g(u), g(v)\}$=1$\forall uv \in E(G_k)$.

Thus G_k admits a Fibonacci prime labeling .Hence G_k is a Fibonacci prime graph.

Example 3.8

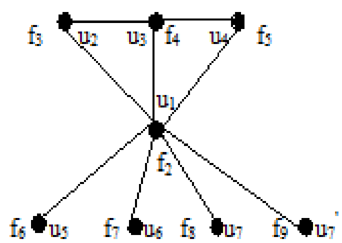

Figure 4: Duplication of a pendant vertex u_8 in O_3 .

Theorem 3.10

The graph obtained by identifying any two pendant vertices u_i and u_k of an octopus graph O_n is a prime graph, where n is any positive integer.

Proof:

Let $V(O_n) = \{u_1, u_2, \ldots \ldots . u_{2n+1}\}$ and

$E(O_n) = \{u_1 u_i \;/\; 2 \leq i \leq 2n + 1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$.

Let G_k be the graph obtained by identifying a pendant vertices u_i and u_k in an octopus graph O_n .Here $|V(G_k)|=2n$,where n and k are both odd or even .

Define a function $g: V(G_k) \rightarrow \{f_2, f_3, \ldots \ldots \ldots \ldots f_{2n+1}\}$ as follows

$$g(u_i) = f_{i+1}, 1 \leq i \leq 2n$$

Then the induced function $g^*: E(G_k) \rightarrow N$ is defined by

$$g^*(uv) = \gcd\{g(u), g(v)\} \forall\, uv \in E(G_k).$$

Clearly vertex labels are distinct and $g^*(uv) = \gcd\{g(u), g(v)\} = 1 \;\forall\, uv \in E(G_k)$

Hence G_k admits a Fibonacci prime labeling .Hence the graph obtained by fusing any two pendant vertices u_i and u_k of an octopus graph O_n is a Fibonacci prime graph.

Example 3.11

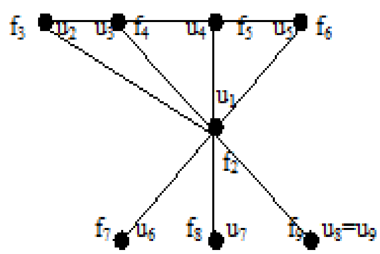

Figure 5:Fibonacci prime labeling by fusing u_8 and u_9

Theorem 3.12

The switching of an apex vertex u_1 in an octopus graph O_n, produces a Fibonacci prime graph, where n is any positive integer.

Proof:

Let $V(O_n) = \{u_1, u_2, \ldots \ldots . u_{2n+1}\}$ and

$E(O_n) = \{u_1 u_i \,/\; 2 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$. Let G_u be the graph obtained by switching an apex vertex u_1 in an octopus graph O_n .Here $|V(G_k)|=2n+1$.

Define a function $g: V(G_u) \rightarrow \{f_2, f_3, \ldots \ldots \ldots \ldots f_{2n+2}\}$ as follows

$$g(u_i) = f_{i+1}, 1 \le i \le 2n + 1$$

Then the induced function $g^*: E(G_u) \to N$ is defined by

$$g^*(uv) = \gcd\{g(u), g(v)\} \forall\, uv \in E(G_k).$$

Clearly vertex labels are distinct and

$$g^*(uv) = \gcd\{g(u), g(v)\} = 1 \,\forall\, uv \in E(G_u).$$

Hence G_u admits a Fibonacci prime labeling .Hence the graph obtained by switching an apex vertex in an octopus graph O_n is a Fibonacci prime graph.

Example 3.13

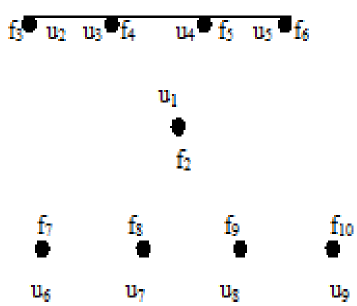

Figure 6: Switching of an apex vertex u_1 in O_4

CONCLUSION

We proved that udukkai graph and octopus graph are all Fibonacci prime graphs. Extending the graph by the operations duplication, switching, fusing are also discussed. Further discussion will be performed in this context.

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